Application of CVaR (Conditional Value at Risk) Analysis In Wind Farm Layout Optimization

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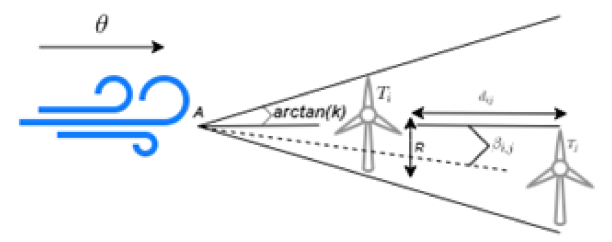
Abstract

This study proposes the optimal layout of an offshore wind farm (WF) allocation for a number (N) of wind turbines (WTs) in a 2 km 2 km fixed area. The paper aims to maximize expected WF power output and efficiency considering the joint probability distribution of wind speed and direction. In fact, the effects of both stochastic variables (wind speed and direction) are considered to find optimal WF layout. Unlike most previous studies using the coordinate model (CM) allows the WTs to be located on any available spot in the WF instead of only at the center of the grids. Thus, by applying joint probability distribution to the WF expected power, a new multivariant conditional value-at-risk model is being presented to find the best possible layout under the worst-case scenarios for both wind speeds and directions. Indeed, the presented optimization model obtains the optimal WF locations.

Wake Effect Model

Although, the Katic wake model has been widely used by researchers, the Jensens model is still the most accepted and fastest models to



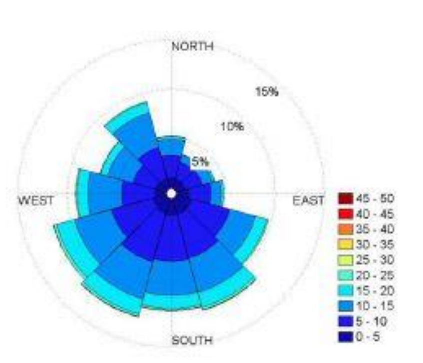
find the wind velocity decay. The wake decay generated by the turbine i located at point T_i is being modeled as a conic shape. According to the following figure, wake effect of turbine i on turbine j can be formulized as follows:

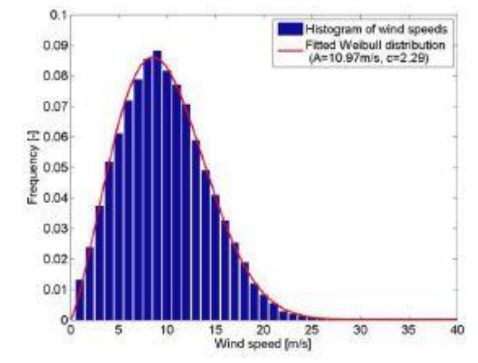
$$v_j^{def} = \sqrt{\sum_{i=1, i\neq j, \Psi_{i,j}<\arctan(k)}^{n} \left[\frac{1-\sqrt{1-C_T}}{\left(1+(k/R)\,d_{i,j}\right)^2}\right]^2}$$

$$d_{i,j} = \left|(x_i - x_j)\cos\theta + (y_i - y_j)\sin\theta\right|$$

CVaR

Due to the uncertainties of the wind, the market loss ranges from 30% to 50% has always been estimated. Even with the best prediction tools, the wind farm owners have been suffered from this uncertainty. Although VaR suffers from lack of sub-additivity and convexity, using CVaR allows us to apply it to the linear programming optimization algorithms. Therefore, finding the expected value of the worst $(1-\alpha)$ cases scenarios of profits is being calculated by a more conservative risk measurement method. Plus, with the coherence of the CVaR method, the risk measurement has preserved the convexity of the robust optimization (RO) counterpart to the optimization problem with the uncertain data, which is also tractable. Also for the uncertain values of wind (direction and angel) we have used the wind rose and Weibull distribution as the below figure :





Wind Farm Efficiency

The efficiency term for a WF is indicated as the percentage of the total power output of all the WTs in the WF produced under the wake effect over the amount of power generation in a free stream without the wake decay (theoretically). In this study, we assumed various wind scenarios, which produces different efficiencies, which can be compared with and without the CVaR to prove the application of our proposed algorithm.
It can be formulated as:

$$\phi_{WF} = \sum_{s=1}^{S_v} f(v) \sum_{i-1}^{N} \frac{1}{N}\frac{P_i}{P_o}$$

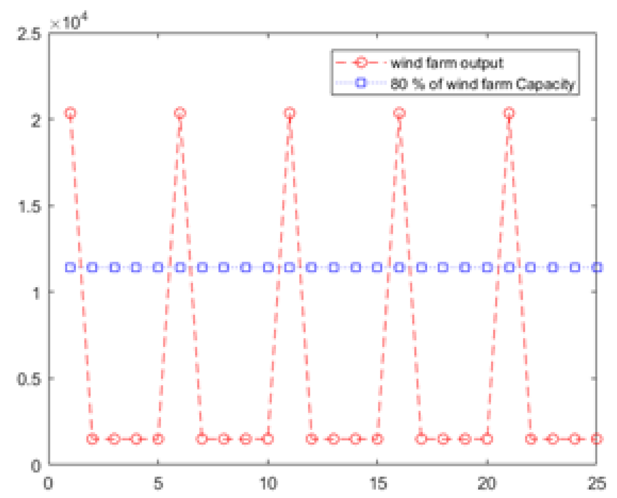
Results and Conclusion

A new CVaR optimization model is presented to find the best possible WF layout under the worst-case scenarios for both wind speed and direction. Using joint probability distribution of these two stochastic variables, a new layout optimization problem is proposed. The aim of the proposed method is obtain WT coordination such that the output power of WF is more than 80% of installed capacity for 95% of possible scenarios. The results show the robustness of the proposed model comparing to non-CVaR models that use the expected WF power as objective function.

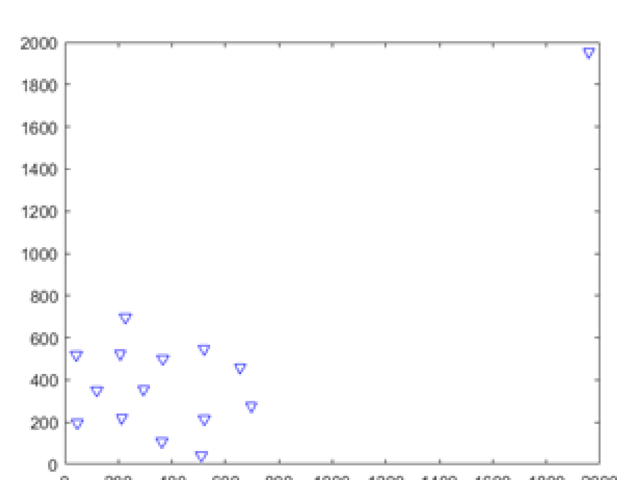
When using the CVaR, one should understand that we are developing a situation where the WF can harness as much power output as possible under the problems conditions. However, it is clear that without using CVaR models, only in five scenarios WF output is over the 80% of the WF capacity. In another word, among the 25 scenarios of WF allocations, five WF micro-sittings can produce enough power to be above the blue line, which shows the 0.8 of the WF capacity.

CHARACTERISTICS OF THE 5 MW WIND TURBINES

Parameters	Value
Hub height, Z(m)	85
WT rotor radius, r_d (m)	63
Downstream rotor radius, r_1 (m)	77.06
the axial induction of WTs, a	0.249
Roughness of the ground, Z_0	0.001
Thrust coefficient, C_T	0.7486
The entrainment constant, α	0.044

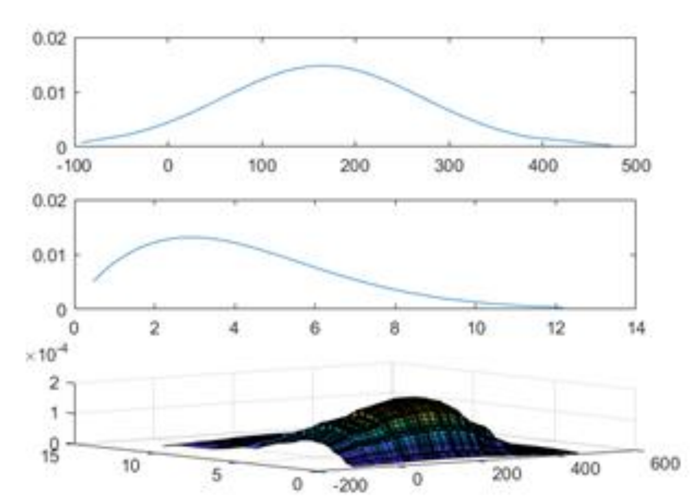


Output power of wind farm for 25 scenarios without using joint probability CVaR optimization



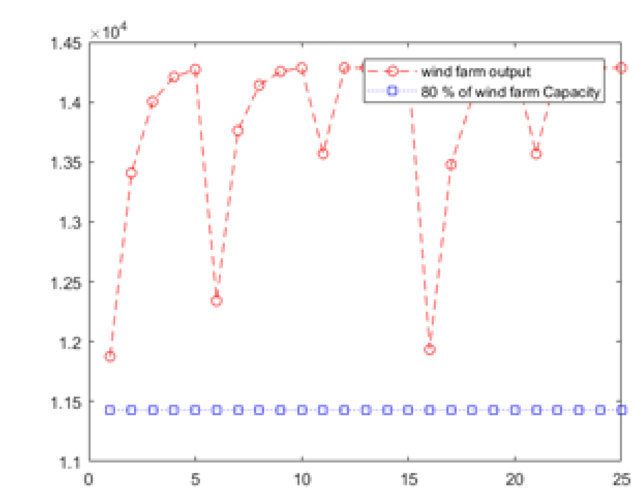
Wind farm layout without considering the joint probability CVar

Distribution of: wind speed, direction and joint probability



Applying the CVaR model

This figure shows that the WF layout is not affected by the wind speed or direction at all. In fact, there is no difference if WTs are placed at down left corner or up right corner. From the optimization point of view, without considering CVaR, the proposed layout in Fig.5 gives the maximum expected output for WF. In other word, the WF layout is not affected by uncertain variables PDF.

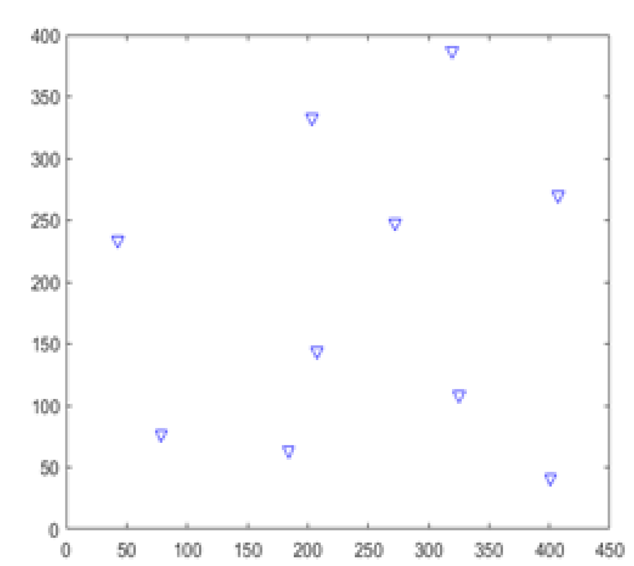


Output power of wind farm for 25 scenarios using joint probability CVaR optimization

It is obvious that the entire 25 scenarios fit into the minimum 80% of the WF capacity. In other word, with 95% confidence level, in all scenarios the output power of WF is more than 80% of installed capacity. Moreover, not only the number of the scenarios to aggregate the maximum power output has been developed, also the average annual energy capacity in this case is more than the previous results.

EFFICIENCIES OF WF UNDER THE DIFFERENT SCENARIOS USING CVAR

Scenarios for θ	θ_1	θ_2	θ_3	θ_4	θ_5
Efficiency %	23.45	23.86	24.67	23.52	24.67



Wind farm layout considering the joint probability CVar

It is also clear that WTs are scattered along with the mean of wind direction distribution, which is 165 degrees. Moreover, with this algorithm, the WF owner needs to invest less amount of the ground to harness the power output. Particularly, with the proposed smart algorithm, the principle investor (PI) can locate the same number of the WTs in the one-twentieth (0.05 of the case study WF) area of the farm.

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