

MATHEMATICS IN EARLY CHILDHOOD:  
AN INVESTIGATION OF MATHEMATICS SKILLS IN PRESCHOOL AND  
KINDERGARTEN STUDENTS

BY

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## Abstract

The current literature in early childhood mathematics provides for little explanation of early mathematics skill acquisition in young children. This study was designed to use existing research on specific early mathematics skills to examine a cohesive model of mathematics skills in preschool and kindergarten aged students. Preschool and kindergarten students were recruited from several programs in New York State, and each of these students completed a set of eight assessment tasks measuring aspects of early mathematics skills. Results of confirmatory factor analyses indicated that combining the assessment tasks already demonstrated in the literature provided an appropriate model of early childhood mathematics skill acquisition. Seven skills demonstrated moderate to strong correlations with a latent variable of early childhood mathematics skills: One to one correspondence, the stable order principle, cardinality, the order irrelevance principle, number identification, oral counting, and informal addition and subtraction. The number skill of magnitude demonstrated a weak correlation with early childhood mathematics skills in this study. The variables that demonstrated the strongest correlations were informal addition and subtraction and the stable order principle followed closely by cardinality, one to one correspondence, order irrelevance, number identification, and oral counting skills. This information can be used to guide assessments for this age group, which often limit which of these areas are assessed. In addition, knowledge about the relationships of specific tasks to overall skill development can guide Response to Intervention practices where teachers assess children's levels of learning and develop instruction specific to those needs.

## Chapter 1: Introduction

Within American education, there is an increasing emphasis on children being able to meet the national standards set in the federal No Child Left Behind Act. The overall goal of this Act is to close the achievement gap between disadvantaged and minority students from the more advantaged and majority students. This Act states that all children will demonstrate measurable gains in reading and mathematics until they are performing at or above grade level in these subjects. While the emphasis is currently on reading acquisition, this law mandates that children will have to meet the standards for mathematics by the 2013-2014 school year (Education Commission of the States, 2002).

Furthermore, there is a trend in both federal and state legislation regarding Response to Intervention. The Individuals with Disabilities Education Improvement Act (IDEA) of 2004 (34 CFR § 300, 2006) has made federal changes regarding the policy on the classification of students with specific learning disabilities. Section 300.307 has been revised to state that, “The criteria adopted by the State must permit the use of a process based on the child’s response to scientific, research-based interventions but cannot prohibit the use of the severe discrepancy model to classify a student with a specific learning disability.” This means that although students can still be identified as a student with a disability by demonstrating a significant discrepancy between their cognitive ability and academic achievement, states may also consider a student’s history of their responses to research-based interventions as criteria for classification.

The current trend in early childhood education includes developing instructional methods that account for both general developmental abilities and unique child characteristics. At the preschool and kindergarten levels, children tend to develop academic abilities at

various rates when compared to their same-aged peers (Bowman, Donovan, & Burns, 2000). Minding these differences in acquisition, previous research has set the standards for reading and mathematics instruction, placing the development of concepts as a necessary precursor to more difficult skill acquisition. In other words, the development of these basic underlying concepts serves as the basis for later learning and achievement within reading and mathematics (Anthony & Lonigan, 2004; Campbell, Pungello, Miller-Johnson, Burchinal, & Ramey, 2001; Chaney, 1998; Gelman & Meck, 1983; Lonigan, Burgess, Anthony, & Barker, 1998; Resnick, 1989; Stevenson & Newman, 1986; Whitehurst & Lonigan, 2001)

A well-developed foundation of research exists within the area of emergent and early literacy that leads to an appropriate application of the Response to Intervention trend. This research has defined the pre-reading skills that are important for later reading development. The early skills that are currently believed to be the most important to preschool reading development are phonological sensitivity/awareness, oral language skills, and an understanding of the conventions of print (Anthony & Lonigan, 2004; Chaney, 1998; Lonigan, Burgess, Anthony, & Barker, 1998; Whitehurst & Lonigan, 2001). These skills may be best conceptualized as a continuum upon which emergent reading skills begin and later literacy skills emerge, with no clear beginning or ending points (Lonigan, Burgess, & Anthony, 2000).

From the information obtained by researchers in the area of reading, some curriculum-based measures have been developed to screen skill acquisition in the various areas of early literacy development. These measures meet the criteria of the Response to Intervention model and are appropriate for early intervention with students. One such

measure is the Dynamic Indicators of Basic Early Literacy Skills (DIBELS). The DIBELS were developed by researchers Good and Kaminski (1996) and were based on some of the curriculum based assessment research completed by researchers Deno (1985), Deno and Fuchs (1987) Deno and Mirkin (1977), and Shinn (1989). The result is a set of quick, one minute fluency tasks to measure and monitor early literacy and reading skills in children grades Kindergarten through Six. Schools are able to interpret performance on these tasks in order to make decisions regarding individual or group progress and the educational instruction of students (Coyne & Harn, 2006; University of Oregon Center on Teaching and Learning, n.d.).

While the research in the area of early literacy is fairly comprehensive, the research in the area of early numeracy is lacking. Limited research is available in this subject area. However, although not nearly as clear, the research in the area of mathematics development has also defined important skills for the foundation of later math achievement. These skills tend to fall within two main categories: counting concepts and number sense (Campbell et al., 2001; Gelman & Meck, 1983; Resnick, 1989). Within the literature, these skills have been shown to predict later mathematics scores. For instance, in one study by Locuniak and Jordan (2008), kindergarten students were administered number sense activities with regard to counting, number knowledge, nonverbal calculation, story problems, and number combinations. Each of these areas was shown to predict later math calculation skills in second grade. In another study completed by Stevenson and Newman (1986), preschoolers were administered the Arithmetic subtest of the Wide Range Achievement Test (WRAT). This subtest consists of counting and recognizing and reading number symbols. When completing a

longitudinal comparison, the students' scores on these tasks had a .46 correlation with math achievement on the same test in tenth grade, which at this grade level, includes oral and written mathematics computation tasks. When looking at problem solving skills measured by tasks designed through the use of a mathematics curriculum, this correlation with first grade WRAT scores is .52. Although these are moderate correlations, there appears to be a relationship between the early understanding of mathematical concepts and later achievement, even into high school. Finally, the National Mathematics Advisory Council also completed research to identify the appropriate skills needed early in mathematics education to be successful in high school algebra (National Mathematics Advisory Panel, 2008).

With regard to counting concepts, counting can be thought of as using words to count and represent numbers. In order to be able to count, children must learn the underlying concepts of counting. These counting concepts include five main principles: one-to-one correspondence, the stable order principle, cardinality, the order irrelevance principle, and the abstraction principle (Gelman & Meck, 1983). One-to-one correspondence means that when counting, each object has one unique number word, and the stable order principle maintains that these number words must remain in the same order each time the child counts. Cardinality refers to when counting, the last number word counted represents the total number of objects in the counted set. The order-irrelevance principle states that when counting a set of objects, the order in which they are counted is not important as long as all of the objects are counted. Finally, the abstraction principle implies that when counting any set of objects, all of the other four principles must apply. For example, if a child were counting a group of animals and a

different group of dots, he would count each group of objects by following the four basic principles (Briars & Siegler, 1984; Gelman & Meck, 1983).

According to the literature, number sense is characterized by the acquisition of skills rather than the understanding of principles. Although this area of research is still emerging, some skills associated with number sense have been shown to be developed in preschool students. These skills include number identification, counting (the skill rather than understanding the basic principles as presented above), understanding the magnitude of objects, and the ability to complete informal addition and subtraction tasks. Number Identification includes making a connection between a written number symbol and the number word, while counting is the process of using words to count and represent numbers (Clarke & Shinn, 2004; Gelman & Meck, 1983; Gersten, Jordan, & Flojo, 2005). Magnitude includes the ability to look at a set of objects and discriminate what amount is the larger amount, regardless of how information is presented (Bertelli, Joanni, & Martlew, 1998; Murray & Mayer, 1988; Sophian, 2000). Finally, informal addition and subtraction tasks include the use of protoquantitative comparisons and understanding increases or decreases in quantities. That is, children can understand that adding objects to an array of other objects results in more, while removing objects results in less (Klein & Bisanz, 2000; Resnick, 1989). Children begin to learn and acquire these skills during preschool and kindergarten, prior to receiving a significant amount of formal instruction in mathematics. Even in preschool and beginning kindergarten, children are gaining an understanding of number concepts through activities with their friends and families, with little or no formal mathematics training (Campbell et al., 2001).

Thus, there is considerable empirical evidence regarding which skills need to be developed in order to increase learning and achievement in the areas of mathematics and reading, and links to later academic achievement have been demonstrated. However, there has been little research linking early mathematics concepts to one another in order to be considered a model of overall early childhood mathematics skill development. Given the research linking early skill acquisition to later academic achievement, it is imperative that early mathematics skill development be well defined in order to adjust the curriculum standards for young children and aid in the application of Response to Intervention methods.

This information leads to the logical proposal of research in the area of early childhood mathematics. Changes in instructional methods would assist educators in meeting the national standards for mathematics, and perhaps more importantly, may lead to earlier detection of skill deficits to be targeted by research-based interventions. While the research in early reading skills is well-defined, there is a lack of research regarding the continuum of emergent math skills and tasks to measure those skills. Therefore, this research study focuses upon early mathematics development in preschool and kindergarten students. The literature has already identified some areas of early math development, which in this research will be considered counting concepts and number sense. The aim of this research was to take the previous literature one step farther and identify a cohesive model of emergent numeracy that begins to explain early mathematics development in preschool and kindergarten children. Based upon the previous literature, hypotheses in the present study included:

1. Measures of the principles of One-to-One Correspondence, Stable Order, Cardinality, and Order Irrelevance will correlate to measure the overall construct of Counting Concepts, as supported by literature by Gelman and Meck (1983).
2. Measures of Number Identification, Counting, Magnitude, and Informal Addition and Subtraction will correlate to measure the overall construct of Number Sense.
3. The constructs of Counting Concepts and Number Sense will correlate to create a larger variable of Early Childhood Mathematics Skills.

## Chapter 2: Method

### *Participants*

Participants were recruited for this study from 10 preschool and four kindergarten classrooms in the Southern Tier of New York State and Central New York. Consent forms were sent home to parents in these schools. This form explained the study to parents and also asked for a signed consent form with information regarding the student's name, school, date of birth, sex, the number of years spent in preschool, and the mother's level of education. This form can be viewed in Appendix A. The resulting participants for this study included 88 students who were already five-years-old or who would turn five-years-old by December 1, 2009. Of these recruited students, 47 were participating in preschool programs while 41 were in kindergarten classrooms. These student ranged in age from 54 to 72 months old, with the mean age of participants being 61.6 months old. Thirty-seven of those participants were female and fifty-one were male. The average number of years spent in preschool was 1.3 years, and the average level of education by the participants' mothers was achieving an Associate's Degree. This information is included in Table 1.

### *Model*

This study aimed to develop a model of early mathematics development in preschool and kindergarten children, similar to that which exists in the area of early literacy. The original model consisted of one overall construct of Early Childhood Mathematics Skills that could be divided into two components: Counting Concepts and Number Sense. The variables that comprised the Counting Concepts construct were based upon research already completed by researchers Gelman and Meck (1983). The variables

comprising the Number Sense construct were others supported by early mathematics literature (Bertelli et al., 1998; Clarke & Shinn, 2004; Gelman & Meck, 1983; Gersten et al., 2005; Klein & Bisanz, 2000; Murray & Mayer, 1988; Sophian, 2000). This hypothesized model can be viewed in Figure 1.

Counting Concepts can be considered the combination of five main principles that must be understood in order to develop counting skills: the One-to-One Correspondence Principle, the Stable Order Principle, the Cardinality Principle, and the Order Irrelevance and Abstraction Principles (Gelman & Meck, 1983). The One-to-One Correspondence Principle states that each number, when counted, has its own unique tag. The Stable Order Principle means that those tags are always used within the same order. The Cardinality Principle refers to the last number counted representing the total number of objects in a set. The last two principles, Order Irrelevance and Abstraction mean that objects can be counted in any order, and anything can be counted, so long as all of the other rules are followed.

For the purpose of this study, Number Sense is considered to consist of the mathematics skills that four- and five-year-old students are able to master prior to completing Kindergarten. The first variable in this construct is Number Identification which is recognizing numbers as symbols and connecting that symbol with a label (Clarke & Shinn, 2004; Gersten et al., 2005). The second is Counting, which includes using words for counting and representing numbers (Gelman & Meck, 1983). The third variable is Magnitude, which can be defined as understanding aggregate amount and discriminating among different sizes (Bertelli et al., 1998; Murray & Mayer, 1988; Sophian, 2000). Finally, the fourth variable of this construct is Informal Addition and

Subtraction. This variable measures a child's understanding of the concepts of addition and subtraction, without actually having learned the operation (Klien & Bisanz, 2000).

### *Measures*

This study was divided into three overall construct measures: Early Childhood Mathematics Skills, Counting Concepts, and Number Sense. In order to measure the variables comprising these construct measures, assessments were adapted from those in the current literature. Participants were individually administered nine tasks to measure these variables in one sitting. In order to measure Counting Concepts, participants completed measures of the One-to-One Correspondence Principle, the Stable Order Principle, the Cardinality Principle, and the Order Irrelevance Principle. In order to measure Number Sense, the participants completed a measure of Number Identification, two measures of Counting, a measure of Magnitude, and one measure regarding Informal Addition and Subtraction. The approximate time for administration was between 15 and 25 minutes for each child. Details regarding these measures are described in this section.

### *Counting Concepts*

Children's understanding of Gelman and Meck's (1983) counting principles were assessed. The One-to-One Correspondence Principle, the Stable Order Principle, and the Cardinality Principle were measured by modifying the methodologies used by Gelman and Meck. These measures were recognition tasks rather than measures of the child's ability to demonstrate the principle on their own. These researchers created the measures as recognition tasks in order to ensure that they were measuring the child's understanding of the principle, not the ability to generate and monitor their own counting performance, a difficult task for the level of cognitive development associated with this age group

(Gelman & Meck, 1983). The Order Irrelevance Principle was assessed by utilizing a modified version of the methodologies used by Briars and Siegler (1984).

These tests were modified from their original versions in a few ways. First, the total numbers of questions were decreased. Because a majority of children in the original studies were able to identify the errors and correct trials with accuracy, the numbers of trials were decreased for the current study. Second, a smaller number of set sizes were used for the same reason and also to attempt to decrease the amount of time required for this task to be completed while maintaining validity. Specific changes will be discussed in the following section describing each of the individual assessments.

*One-to-One Correspondence Principle.* The One-to-One Correspondence task included the use of a puppet counting a linear array of red and blue blocks. During this task, the puppet counted some of the trials correctly, and others were in error. In the correct trials, a puppet correctly counted a linear array of objects from beginning to end. During the in-error trials, the puppet either skipped or double counted an object.

To begin the experiment, the children were shown an array of red and blue blocks. They were then told: “This is my friend Mr. Lion. He would like you to help him play a game. Mr. Lion is going to count the blocks on the table but Mr. Lion is just learning how to count and sometimes he makes mistakes. Sometimes he counts in ways that are OK but sometimes he counts in ways that are not OK and that are wrong. It is your job to tell him after he finishes counting if it was OK to count the way that he did or not OK. So remember, you have to tell him if he counts in a way that is OK or in a way that is not OK and wrong” (Gelman & Meck, 1983).

In the original one-to-one correspondence study, participants were administered four trials per set size including two correct and two error trials for sets of 6, 8, 12, and 20 items. In the current study, participants were administered four trials with two correct and two incorrect counts for set sizes of 6 and 12 items. For each set size, one trial was correct and one was in error. The children's answers were recorded, and the number answered correctly was measured. The directions and materials for this test can be viewed in Appendix B.

*Stable Order Principle.* For the Stable Order Principle, several small objects were set up in a row of 5, 7, 12, or, 20 pieces. These objects were foam shapes of different colors (hearts, circles, squares, flowers, and stars) and foam sports shapes (basketballs, soccer balls, baseballs, baseball bats, baseball hats, volleyballs and footballs). In the original study by Gelman and Meck (1983), the children were administered five trials in each of the set sizes of 5, 7, 12, and 20 items. Two of those trials were correct counts while three were in error. For the current study, each child was administered four trials. In two of the trials, the same puppet as used in the One-to-One Correspondence Principle task counted the objects correctly, and in the other two, a violation of the Stable Order Principle occurred. The violations included reversing the order of items (e.g. 1, 2, 3, 5, 4, 6) or skipping one or more tags in a sequence (e.g. 1, 2, 3, 5, 6). Set size and error type were randomized according to each of the original study's set sizes. The children's responses were recorded in order to measure the number of items answered correctly. Details for this measure can be viewed in Appendix C.

*Cardinality Principle.* In order to measure the Cardinality Principle, new directions were given based upon the original study by Gelman and Meck (1983).

Children were asked to state whether the puppet gave the correct answer to “How many?” after counting the objects. The set sizes and objects were the same as in the stable order tasks and modifications from the original study are also the same as in the previous tasks. The children were administered four trials. The puppet responded correctly two times and in-error two times. Again, order and error were randomly chosen, and the children’s answers were recorded to measure correct responses. This measure can be viewed in Appendix D.

*Order Irrelevance Principle.* The methodology from Briars and Siegler (1984) was modified in order to measure the Order Irrelevance Principle. In the Briars and Siegler study (1984), children were administered rows of 3, 4, 9, and 10 chips pasted on a cardboard strip. They encountered counting errors, unusual correct counts, and standard correct counts as demonstrated by a puppet. The children in this age group were able to effectively demonstrate these skills on the set size of ten, which was then used in this current study. Specifically, one row of ten wooden chips were pasted one half inch apart on a cardboard strip and presented to the children. The chips alternated in color in order to maximize the likelihood that children were able to distinguish the counted from the uncounted chips. A puppet counted the chips for the child. Seven questions were administered to the children including three counting errors, one standard count, and three unusual counts. The three counting errors included violating word/object correspondence by assigning too many number words to one of the objects, omitting a word by pointing to an object but not saying anything, or skipping objects altogether. Three types of unusual counts included counting from right to left, starting in the middle of the row and finishing on the other side of that object, and counting every other object

until they reached the end of the array and then reversing the counting back the opposite way to complete counting the objects that had not yet been counted. The one standard type of counting included correctly pointing to and counting every object from left to right. These trials were randomly organized.

Children began this task by being introduced to Callie Cat, a new puppet. They were told, “Callie Cat is going to count for you. Callie Cat is just learning how to count. Sometimes when she counts, she makes mistakes. She knows her numbers, but sometimes when she counts the chips, she does it wrong. Callie Cat is going to do some counting for you. I want you to watch her very carefully to see if she makes a mistake. Let’s try one for practice. Watch carefully.” Callie Cat then counted a row of ten chips, skipping three adjacent chips in the middle of the row and one at the end. Afterwards, the children were asked, “Did Callie Cat make a mistake?” After answering this correctly, children began the seven test trials (Briars & Siegler, 1984). Again, correct answers were recorded for analysis. This measure can be viewed in further detail in Appendix E.

### *Number Sense*

The Number Sense construct consisted of four variables: Number Identification, Counting, Magnitude, and Informal Addition and Subtraction. Each of these variables was measured by a different task. The Number Identification task was measured by Clarke & Shinn’s (2004) AIMSweb Number Identification benchmark assessment. The counting task was measured by the AIMSweb Oral Counting benchmark assessment (Clarke & Shinn, 2004) as well as by a task where the student looked at pictures of lady bugs and counted the number of dot on the lady bugs’ wings. This measure was designed by the examiner. The Magnitude task was a modified version of that completed by

Sophian (2000). Finally, the Informal Addition and Subtraction task modeled a study completed by Klein & Bisanz (2000). Each of the Magnitude and Informal Addition and Subtraction tasks were modified from their original version with regard to the number of trials administered to each child because the children demonstrated the ability to complete the activity in the original studies and also to reduce the amount of administration time for each of the tasks.

*Number identification.* In order to measure Number Identification, the Fall Kindergarten Benchmark probe for the AIMSweb Number Identification assessment was used (Clarke & Shinn, 2004). This probe is a sheet of randomly generated numbers from one to ten. These numbers were copied on a separate card to be used for this assessment. Children were given a copy of the probe and asked to read aloud as many numbers as possible in 60 seconds. As the child read the numbers aloud, the examiner recorded any errors and marked the last number read by the child in 60 seconds. Errors included numbers read incorrectly, skipped numbers, and those not read within three seconds. In order to calculate Number Identification Fluency, the examiner subtracted the numbers read incorrectly from those which were read correctly (Clarke & Shinn, 2004). A full write up of this assessment can be viewed in Appendix F.

*Counting.* The next measure was a test of using words for counting. Children were asked to count in two different ways. The first measure of counting was oral counting, following the procedure outlined in the AIMSweb Oral Counting Task. That is, the children were asked to count out loud without any visual materials. The examiner said to the child, “When I say start I want you to start counting out loud from 1 like this 1, 2, 3 until I tell you to stop. If you come to a number you don’t know, I’ll tell it to you. Be sure

to do your best counting. Are there any questions? Ready, Start” (Clarke & Shinn, 2004). The examiner recorded the child’s responses for 60 seconds and calculated the Oral Counting score by subtracting errors from the highest number counted. Errors included skipping a number or hesitating on a number for three or more seconds. This measure can be viewed in Appendix G. Next, children’s counting skills with regard to one-to-one correspondence were assessed. The participants were shown large pictures of lady bugs with different numbers of dots on their wings (three, five, or seven) and asked, “How many dots does this ladybug have on her wings? Point to each dot and count them out loud for me.” The examiner then recorded responses and marked the number correct out of three possibilities. The directions and materials for this measure can be found in Appendix H. Each of these measures was recorded separately for further statistical analysis.

*Magnitude.* In order to measure Magnitude, a version of Sophian’s (2000) methodology was utilized. The number of sets was reduced from the original study from four sets to one, using only circular shapes. The purpose for this was that children in the original study performed better on circular shapes than unusual shapes, and only one set was administered in order to reduce the time of administration. Children were presented with pages of standard paper with different shapes on them, called “cookies.” These pages were placed in a three ring binder so that as the examiner turned the pages, the child was able to compare two pictures to one another. One page, printed in green, always contained either one or two cookies on it, and the other page, printed in magenta, always contained either four or five cookies on it. Students were presented sets of four cookies and two cookies or five cookies and one cookie. The ratios of the aggregate amounts

were either 2.5:1 or 4:1. Students were presented with two types of problems: Amount Agreement and Amount Conflict. In the Amount Agreement trials, materials were presented such that the greatest number of cookies was always the greatest magnitude. In the Amount Conflict trials, students were presented with materials where the greatest number of cookies was always the least total amount.

The procedure for this assessment included an introduction and eight problems. The introduction included the use of the puppet from the previous measure. Children were told that the puppet was hungry from all of that counting, and wanted to fill her belly with cookies. They were told that, “She does not care how many cookies she gets, so long as she gets lots to eat” (Sophian, 2000). They were then asked to help decide which set of cookies was the most, and therefore which ones the puppet should eat. The items consisted of four Amount Agreement trials and four Amount Conflict trials. A full write up of this procedure can be found in Appendix I.

*Informal addition and subtraction.* Finally, in order to assess informal addition and subtraction, the methodology completed by Klein and Bisanz (2000) was modified. This study was modified from the original version by reducing the set size from 12 problems to 8, in order to make adjustments for working memory and using the simplest items for teaching problems. The students in the current study completed only two-term problems as three-term problems were demonstrated to be difficult for this age group in the original study. Children watched an array of colored chips be placed on the table and then covered with a box. The experimenter then added or subtracted chips from the hidden array while the children observed. When the experimenter was finished, the child was asked to match the same number of chips in front of them as was under the box.

After completion, the box was removed and the next problem was presented without feedback on accuracy. The child completed eight two-term single digit problems where half were subtraction and half were addition. The children were also presented with an introduction to each type of task. Children's responses were recorded for accuracy. A full description of this assessment can be found in Appendix J.

### *Procedure*

Four-year-old and five-year-old preschool and kindergarten students were assessed with the previously mentioned measures. The current author, one graduate professor, and four graduate students from the Alfred University School Psychology program were involved in data collection. The graduate students and graduate professor were trained in administration of the measures by the current author. Data were collected over a period of approximately four to five months beginning in late June and ending in early October. Students completed the assessment measures individually with one of the researchers in a location outside of their classroom and with as few distractions as possible. After the teacher introduced the child to the researcher, the researcher took a few moments to gain rapport with the students and make them feel comfortable. Depending on the student, the amount of time of assessment varied from approximately 15 to 25 minutes. When requested, parents were provided with results of the study after data analysis was completed.

*Data analysis.* The hypothesized model used for data analysis can be viewed in Figure 1. Two first order and one second order confirmatory factor analyses were used to examine the data. First, the Counting Concepts construct was analyzed through the use of a confirmatory factor analysis. The purpose of this analysis was to determine if the

measures of One-to-One Correspondence, the Stable Order Principle, the Cardinality Principle, and the Order Irrelevance Principle were correlated to measure one overall construct, stated here as Counting Concepts. Next, the Number Sense construct was analyzed through the use of another confirmatory factor analysis. The purpose of this analysis was to determine if the measures of Number Identification, Counting, Magnitude, and Informal Addition and Subtraction were correlated to measure the overall construct named here as Number Sense. Finally, the two combined constructs of Counting Concepts and Number Sense were then analyzed with a second order confirmatory factor analysis to determine if these two constructs could be determined to be appropriate measures of an overall construct of Early Childhood Mathematics Skills. In addition, the demographic variables of age, number of years in preschool, sex, and socio-economic status were included in this analysis.

The current assessments for this study utilized different scales for the measurement of student mathematics skills. Two variables, Number Identification and Oral Counting, were continuous variables that represented scales much larger than those of the other measures. In order to avoid statistical problems within the analysis, each of these variables was recoded in order to reduce the scale to 8. This change was necessary in order to create more compatible measures for comparison. The Number Identification measure was originally a scale from 0 to 56. This scale was divided by 8 in order to reduce the scale from 56 to 8 categories. A similar method was utilized for Oral Counting. The raw data for this measure provided a scale from 7 to 75. This information was recoded by subtracting 7 from the total and dividing by 8.

### Chapter 3: Results

All data collected from the assessment measures was entered into the Statistical Package for the Social Sciences (SPSS). This information was then analyzed in the Analysis of Moment Structures (Amos) program through several confirmatory factor analyses. The Amos program provides several indices to estimate how well the model fits the data. The indices used in this study included the Chi Square, Root Mean Square Error of Approximation (RMSEA), and the Comparative Fit Index (CFI). When analyzing Chi Square data, a model that has a non-significant Chi Square is considered a good fit to the observed data (Howell, 2002). For the maximum likelihood that the model fits the data, a cutoff value of .06 or lower is considered acceptable for RMSEA and a value of .95 or higher is recommended for the CFI (Hu & Bentler, 1999). A table of the descriptive statistics for all assessment measures can be viewed in Table 2.

The first confirmatory factor analysis completed was for the Counting Concepts construct. This information was analyzed in order to determine if the measures of One-to-One Correspondence, the Stable Order Principle, the Cardinality Principle, and the Order Irrelevance Principle correlated in order to measure the Counting Concepts construct. The analysis of this data can be viewed in Figure 2. When examining the results of this analysis, some important information is evident. Each of the correlations associated with the test measures and the Counting Concepts construct can be considered a moderate (Cardinality Principle, Order Irrelevance Principle, and One-to-One Correspondence Principle) or strong correlation (Stable Order Principle). This suggests that these measures do indeed correlate to create an overall construct, as hypothesized in this study and supported by studies completed by Gelman and Meck (1983). In addition,

the Chi Square value for this model was non-significant ( $\chi^2 = 8.783, p = .889$ ) and RMSEA (.000) and CFI (1.0) values also confirmed that this model is a good fit to the data. It is also important to note that among these variables, the one with the strongest overall correlation and influence on the Counting Concepts construct is the Stable Order Principle ( $r = .78$ ) followed by the Cardinality Principle ( $r = .58$ ), the Order Irrelevance Principle ( $r = .54$ ), and finally, the One-to-One Correspondence Principle ( $r = .48$ ). Each of the demographic variables demonstrated weak correlations with Counting Concepts, suggesting that further analysis of these variables would not lend to further explanation of these results.

Another confirmatory factor analysis was completed on the Number Sense construct. This factor analysis measured the correlations for the measured variables Number Identification, Oral Counting, Magnitude, and Informal Addition and Subtraction with the overall construct of Number Sense. The analysis of this data can be viewed in Figure 3. As evidenced in the analysis of the Counting Concepts construct, most of the measured variables suggested moderate to strong correlations with the Number Sense construct with the exception of the Magnitude variable. Although these correlations appear high, the fit indices do not indicate an appropriate fit to this model. The Chi Square value for this model fit was significant ( $\chi^2 = 30.403, p = .011$ ), and RMSEA (.109) and CFI (.735) values suggested a poor fit between this model and the data. This information did not support the original hypothesis, so further exploration was conducted with these variables.

There was another task included in this study which was originally considered a measure of counting. This task required the students to look at pictures of lady bugs and

to count the number of dots on each of the lady bugs' wings. In order to determine if this task was indeed a measure of counting, a confirmatory factor analysis was completed where the Counting variable was considered a latent variable with measured variables of this Lady Bug task and the AIMSweb Oral Counting Task. This analysis demonstrated a poor fit between the model and the data suggesting that this Lady Bug task is not a measure of traditional counting. While this measure did not fit into the originally hypothesized variable, it was still considered an important variable and further analysis was conducted in order to examine its effects on the model. The same type of confirmatory factor analysis was completed for the One-to-One Correspondence variable included in the Counting Concepts construct. The One-to-One Correspondence variable was made into a latent variable with the Lady Bug task and the Gelman and Meck (1983) One-to-One Correspondence Principle task as measured variables. Again, this analysis demonstrated a poor fit between the model and the data, suggesting that the Lady Bug task does not fit into this area.

Next, a confirmatory factor analysis was conducted in order to assess if the Lady Bug measure was correlated to the overall measure of Counting Concepts. Again, this analysis suggested that the Lady Bug measure did not correlate with this construct either. Finally, the Lady Bug measure was included in a confirmatory factor analysis of the overall construct of Number Sense. This analysis demonstrated a good fit for this model ( $\chi^2 = 22.081, p = .395$ ), with a strong correlation between the Lady Bug measure and Number Sense ( $r = .60$ ). Given this information, the Lady Bug measure was named a One-to-One Correspondence Task, and was included in the overall construct of Number Sense as a single variable. When examining this model with all of these variables,

correlations are considered within the moderate to strong ranges with the exception of the Magnitude variable. The Magnitude Task demonstrated a weak correlation of .14. The variable associated with the strongest correlation was the Informal Addition and Subtraction Task ( $r = .81$ ) followed by the One to One Correspondence Task (.60), the Oral Counting Task ( $r = .52$ ), and then the Number Identification Task ( $r = .50$ ). Again, the demographic variables had weak correlations with Number Sense, so no further analysis in this area was conducted. This model can be viewed in Figure 4.

The next step in the analysis was to complete a second order confirmatory factor analysis of the hypothesized model. This analysis took each of the Counting Concepts and Number Sense constructs and examined their effects on an overall measure of Early Childhood Mathematics Skills. Results of these fit indices indicated that the hypothesized model was inadequate and did not fit the data well. The Chi Square for this model was 135.360 with a probability level of less than .001. The RMSEA and CFI were .112 and .556, respectively. See Figure 5 for this model.

#### *Modifications to the Model*

The second order confirmatory factor analysis of the hypothesized model indicated that this model did not demonstrate a good fit to the overall data. In order to examine alternatives to this model, the modification indices provided by the Amos program were utilized. These indices indicated that some of the measured variables were related to both the Counting Concepts and the Number Sense constructs. In order to examine the correlations of each of the variables with Early Childhood Mathematics Skills, another confirmatory factor analysis was conducted. In this analysis, the Counting Concepts and Number Sense constructs were removed altogether. In addition,

modification indices were again utilized in order to improve the model's fit to the data. Given these suggestions, correlations were drawn between the error variables of the One-to-One Correspondence Principle and the Stable Order Principle, the Stable Order Principle and the Order Irrelevance Principle, the Stable Order Principle and the Addition and Subtraction Task, the Cardinality Principle and the Addition and Subtraction Task, the Order Irrelevance Principle and the Addition and Subtraction Task, and the Number Identification Task and the Oral Counting Task. Correlations were also included among the demographic variables to measure their correlations with one another. These correlations were drawn in as suggested in the modification indices of the Amos software program, which allows these correlations to exist for analysis. The resulting model indicated a good fit to the data. The Chi Square for this model was 43.736 with a probability level of .840. The RMSEA and CFI were .000 and 1.0, respectively. Each of the demographic variables demonstrates a weak correlation with Early Mathematics Skills. This model can be viewed in Figure 6.

When viewing this series of factor analyses, the variables that demonstrate the strongest correlations with Early Childhood Mathematics Skills are the Informal Addition and Subtraction Task ( $r = .81$ ) followed by the Stable Order Principle ( $r = .79$ ). This information suggests that these tasks have the greatest amount of influence on Early Childhood Mathematics Skills. Each of the other tasks, with the exception of the Magnitude Task, demonstrate moderate correlations with Early Childhood Mathematics Skills. These correlations include the Cardinality Principle ( $r = .58$ ), the One-to-One Correspondence Task ( $r = .57$ ), the One-to-One Correspondence Principle ( $r = .55$ ), the Order Irrelevance Principle ( $r = .50$ ), and the Number Identification and Oral Counting

Tasks (each  $r = .47$ ). The Magnitude Task has the least amount of influence on Early Childhood Mathematics Skills ( $r = .12$ ). Weak correlations are indicated between each of the demographic variables (Age, Years in Preschool, Sex, and Socio-Economic Status) and Early Childhood Mathematics Skills (correlations range from .11 to .24). This indicates that the demographic variables do not add any confounds to this study that need to be controlled.

## Chapter 4: Discussion

Recently, there has been research conducted within the area of early childhood mathematics (Bertelli et al., 1998; Clarke & Shinn, 2004; Gelman & Meck, 1983; Gersten et al., 2005; Klein & Bisanz, 2000; Murray & Mayer, 1988; Sophian, 2000) that has identified specific tasks that relate to early math skill acquisition. These tasks can be divided into two areas: Counting concepts and number sense. Counting concepts can be thought of as the conceptual understanding of the basic principles needed to learn to count while number sense can be considered the skills needed to understand and work with numbers. However, no research to date has identified a cohesive model for math skill acquisition in preschool and kindergarten students. The current research study sought to utilize the previous research and combine the areas of counting concepts and number sense into an overall model of Early Childhood Mathematics.

The first hypothesis tested by this study was that the measures of students' conceptual understanding of the One-to-One Correspondence Principle, the Stable Order Principle, the Cardinality Principle, and the Order Irrelevance Principle would correlate to measure the overall construct of Counting Concepts, as originally demonstrated by Gelman and Meck (1983). A confirmatory factor analysis verified that these measures did indeed correlate and that this model demonstrated a strong fit to the data (see Figure 2). In addition, this analysis demonstrated that the Stable Order Principle had the strongest influence on Counting Concepts followed by the Cardinality Principle, the Order Irrelevance Principle, and finally, the One-to-One Correspondence Principle. The Stable Order Principle demonstrates a strong correlation while the three other variables are of moderate strength.

The second hypothesis tested by this study was that the measures of Number Identification, Oral Counting, Magnitude, Informal Addition and Subtraction, and the One to One Correspondence Task would correlate to measure an overall construct of Number Sense. Another confirmatory factor analysis confirmed that these measures correlated to demonstrate a good fit between the model and the data (see Figure 4). This overall model indicates that the Addition and Subtraction task has a strong correlation with Number Sense while the One-to-One Correspondence Task, Oral Counting Task, and Number Identification Task each demonstrated a moderate correlation with Number Sense. The Magnitude Task demonstrated the weakest correlation with the Number Sense construct.

Finally, it was hypothesized that the constructs of Counting Concepts and Number Sense would correlate to create another variable named Early Childhood Mathematics Skills. That is, it was hypothesized that Early Childhood Mathematics Skills would be made up of two types of skills: those related to Counting Concepts and those related to Number Sense. A second order confirmatory factor analysis was completed to test this particular model and a goodness of fit was not achieved. The variables no longer demonstrated the same strength as a component of Early Childhood Mathematics skills as they did in the individual Counting Concepts and Number Sense Constructs (Figure 5).

In order to attempt to find an appropriate model, another option was investigated. An additional confirmatory factor analysis was conducted to determine the effects of each of the individual variables on the overall construct of Early Childhood Mathematics Skills while removing the constructs of Counting Concepts and Number Sense. This model demonstrated a very strong fit to the data, suggesting an appropriate model of

Early Childhood Mathematics Skills (Figure 6). This model indicates that the two strongest correlations with Early Childhood Mathematics Skills are the Addition and Subtraction Task and the Stable Order Principle. All of the other correlations are moderate level correlations except for the Magnitude Task, which demonstrates a weak correlation.

There are some possible hypotheses that can be made to attempt to explain the lack of fit to the original model (Figure 5) and an appropriate fit to the resulting model (Figure 6). First, it is possible that some of the measures in this study are assessing some of the same skills. For example, in order to be able to complete the Informal Addition and Subtraction task, it is likely that students can already demonstrate the counting principles of one to one correspondence, stable order, cardinality, and order irrelevance. If this is the case, this model, which is outlined in a linear fashion, may not be an accurate representation of the order of skill acquisition of these particular skills. A longitudinal analysis of this data would provide more information to test the possible developmental sequences for acquisition of these skills. Second, many researchers utilize the Occam's razor heuristic when guiding their scientific methodology (Stansfield, 2002). This heuristic states that when an individual is interpreting data, the simplest explanation is often the best explanation. This relates to the current research study by supporting the idea that this model may be too complicated when depicted as the second order confirmatory factor analysis and that the simpler explanation with the variables arranged into a first order confirmatory factor analysis may be more accurate.

### *Implications*

The current study proposes a model of Early Childhood Mathematics Skills for preschool and kindergarten level mathematics skill acquisition. It is relevant to the current literature in early mathematics development for several reasons. First, this information suggests that the measures that have the strongest influence on Early Childhood Mathematics Skills are Informal Addition and Subtraction Tasks and the Stable Order Principle. These measures are then closely followed by the Cardinality Principle, the One-to-One Correspondence Task, the One-to-One Correspondence Principle, the Order Irrelevance Principle, the Number Identification and Oral Counting Tasks, and then finally, the Magnitude Task. The Magnitude Task has a weak correlation so will not be discussed further.

These results can be used to help inform the selection of assessments of early mathematics for this age group. One popular assessment is the AIMSweb curriculum-based measurement system (Clarke & Shinn, 2004). The particular tasks from this system that are designed to assess four- and five-year-old students are the Number Identification and Oral Counting measures that were used in this study. While these two measures demonstrated moderate correlations with the overall Early Childhood Mathematics construct, there were several measures that demonstrated stronger correlations. This might suggest that it would be appropriate to use measures of Informal Addition and Subtraction, the Stable Order Principle, the Cardinality Principle, One-to-One Correspondence, and the Order Irrelevance Principle along with these two measures of Number Identification and Oral Counting.

Besides this example of curriculum-based measurement, there are several standardized assessments designed to measure preschool and kindergarten mathematics

skills. Two popular examples of these types of assessments are the Test of Early Mathematics Ability – Third Edition (TEMA-3) and the Wechsler Individual Achievement Test – Third Edition (WIAT-III). The TEMA-3 was developed by mathematics researchers Ginsburg and Baroody (2003). Four-year-old children begin this assessment by answering questions regarding the principle of cardinality and demonstrating the skill of oral counting. Five-year-old students are asked to write numbers, complete addition and subtraction tasks orally, and demonstrate an understanding of part-whole relationships and the concepts of less and more. The recently revised WIAT-III (Pearson, 2009) contains two specific subtests designed to measure early mathematics skills: Math Problem Solving and Numerical Operations. The Math Problem Solving subtest for this age group measures knowledge of shapes, counting, number identification, and the understanding of the concepts of less and more. The Numerical Operations subtest for this age group measures the principle of cardinality, number writing, and the skills of discriminating numbers from letters.

The current study demonstrates a model for early mathematics skills established through factor analyses. This model indicates that examiners should consider using alternative methods of assessment for this age group instead of relying solely on standardized assessments and curriculum based models. While each of the types of assessments described in this section measure some of the skills demonstrated in the current study, none of them measure all of the skills found to be important in this research. Specifically, the results of this study would suggest that assessment of this age group start with measuring informal addition and subtraction skills and the stable order principle. These two measures demonstrated the greatest correlation strength with overall

early childhood mathematics skills, which would suggest that the other skills have less influence on math skill acquisition at this age. Next, you would assess students who were unable to complete the addition and subtraction and stable order tasks effectively by administering the other measures of cardinality, the one to one correspondence performance task, the one to one correspondence recognition task, order irrelevance, number identification and oral counting in order to find the students' instructional levels. Using this model of assessment, an examiner is able to gain insight into specific skill acquisition.

Furthermore, the research findings are relevant to current teacher practices and can be utilized in several ways. The information gained from this study ties in well to the Response to Intervention model that is now a required teaching methodology to be utilized for all elementary level students (8 NYCRR §100.2, 2007). A summary of this law states that teachers are required to provide appropriate instruction to all students at their particular skill level, with increasingly intensive levels of intervention being provided to students in order to achieve the standards for each academic skill.

In line with the Response to Intervention model, these analyses would first be used in order to design quick and reliable assessment measures for preschool and kindergarten students. The purpose of this type of assessment would be to provide teachers with knowledge regarding students' math skills in a relatively short period of time. This information can then be used to tailor a child's specific learning program as well as the teacher's overall teaching strategies in order to develop students' math skills while in the early grade levels. By targeting teaching methods and individualized interventions, it is likely that gaps in achievement that become evident later in students'

learning might be able to be reduced to a level where these students are able to learn at the same rate as their peers.

#### *Limitations and Ideas for Future Research*

The present study used a relatively small sample of students (N = 88). Due to this small sample size, caution is recommended regarding the generalization of these results. It is recommended that further research with a larger sample of young children be conducted in order to try to replicate these results for greater generalization. The current study recruited students from preschool and kindergarten programs. Given that preschool programs are expensive and unaffordable for some parents, a gap may exist with regard to the socio-economic-status of the current participants. This gap may also be evident for parents who choose to keep their children home from Kindergarten programs until they are of compulsory school age. More information could be obtained from this research if it were replicated with a larger sample size that included greater variance in socio-economic status, race, and location type (rural, suburban, urban settings).

In addition, the demographics questionnaire has a significant limitation that warrants discussion. The parents were asked how many years that their child attended a preschool program. It became evident that while the parents were responding to this question, they were including both day care and preschool programs as the same type of classroom. For example, some parents responded that the children were in a preschool program for four years. Given that children are not eligible for publicly funded preschool services until the age of four, many responses are inaccurate to answer this particular question. The importance of discriminating between the two types of programs is in regard to the educational aspect of the two types of services. Day care programs typically

do not include an instructional component to their service. However, preschool programs are tailored to target school readiness skills through specific curricular models. In the future, it might be important to distinguish the difference between years in a day care setting versus years in a preschool setting.

Another limitation of the current study is with regard to the selection and development of assessment measures for mathematics skills. The only measures that have undergone a study of reliability and validity are those associated with the AIMSweb program. The measures of Number Identification and Oral Counting are standardized measures associated with this program (Clarke & Shinn, 2004). All of the other assessment measures utilized by the current study were modified or developed by the present author.

The One-to-One Correspondence Principle, Stable Order Principle, and Cardinality Principle tasks were modeled from those completed by Gelman and Meck (1983). The Order Irrelevance Principle measure was modeled from research completed by Briars and Sigler (1984). These tests were modified from their original versions by decreasing the total number of questions administered to children and decreasing the set sizes. The Magnitude Task was originally developed by Sophian (2000) while the Informal Addition and Subtraction Task was first completed by researchers Klein and Bisanz (2000). These two tasks were modified with regard to the number of trials completed by the children. In addition, each of these six assessments was originally tested on a small sample of children, and in some cases, has not been replicated until the present study. Finally, the One-to-One Correspondence performance task was developed by the current examiner as a direct measure of students' demonstration of one-to-one

correspondence. While this test was designed with the guidance of Gelman and Meck's (1983) description of one-to-one correspondence, no other study has employed use of this exact measure.

Furthermore, during administration, it was evident that many of the children had a limited attention span or low working memory abilities. Although each of the data collectors involved in this study made their best attempt to keep the children engaged, given the children's age level and cognitive development, some were unable to remain focused for extended periods of time to complete each task. For example, some of the items included the children attending while the puppets counted up to 20 objects. Occasionally, it was observed that the children would lose interest or forget if a mistake was made early on in the demonstration. This varied from student to student, and given that this was observed by each of the administrators, it should be considered a limitation in this study. This limitation has also been demonstrated in other mathematics literature. Specifically, researchers Klein and Bisanz (2000) found that although children are often able to complete mathematical tasks, their working memory skills sometimes impede their ability to demonstrate that conceptual knowledge.

The results and limitations of the current study suggest several areas for future research. First, as mentioned previously, the current research should be replicated with a larger sample size to increase generalization of the results. This study had a relatively small sample size ( $N = 88$ ), and it is possible that a full range of socio-economic status and participant location is not represented in this sample. Some of the students were from urban settings, but these settings are considered small cities, and it would be important to consider differences with larger cities as well as more suburban and rural

areas. Also, there is little variation among racial and ethnic groups represented in this sample, which ideally, could be better represented in a larger sample of participants.

Next, the use of this data should be included in a longitudinal study of mathematics outcomes. It is important to follow up regarding how the skills measured by this particular assessment relate to a criterion measure in each grade level beginning in first grade through the students' educational careers. For example, one would be able to follow up with the participants in this study with a group assessment such as the Group Mathematics Assessment and Diagnostic Evaluation (GMADE, 2008) or standardized state mathematics assessments. The purpose of this follow-up assessment would be to determine the effects of early skill acquisition in the areas measured in this study with later mathematical performance. This information could provide some insight into which of the variables show the greatest level of influence on mathematical performance across grade levels.

Also, the results of this study might indicate that there is more than one way to interpret this data. The first way is how it has been presented until this point. That is, each of the skills measured in this study is correlated to overall Early Childhood Mathematics Skills as indicated in Figure 6. The other way to consider looking at this data is by separating the two constructs of Counting Concepts and Number Sense. Each of these constructs, when analyzed by a confirmatory factor analysis, demonstrated an appropriate fit to the data. It was when those two constructs were compiled to measure Early Childhood Mathematics Skills that it no longer demonstrated a good fit. When considering Counting Concepts and Number Sense separately, the chi square, RMSEA and CFI values are strong for each of these models. If the students from this study are

followed in a longitudinal study, this information would be important to determine the best way to conceptualize Early Childhood Mathematics Skills: as one overall construct, or as two separate ones.

Finally, there is a great deal of research regarding the comorbidity of reading and mathematics disabilities. Many researchers have found that these two types of disabilities are connected and share some of the same skill deficits (Fuchs & Fuchs, 2002; Fuchs, Fuchs, & Prentice, 2004; Kulak, 1993; Light & DeFries, 1995; Rasanen & Ahonen, 1995). There is not very much literature that has investigated the correlations among reading and mathematics skills at the preschool and kindergarten level. Given that many researchers have found commonalities between reading and mathematics disabilities, it is likely that some of the early reading skills and mathematics skills are correlated. For example, many of the early skills in math and reading acquisition are similar. For both math and reading, the concept of one to one correspondence is necessary to understand that each number or letter has one unique tag. Next, number identification and letter identification are similar skills. The students need to learn that each letter and each number has one word that is associated with a unique symbol. Furthermore, when combining numbers, children begin to understand larger and more complex numbers. This skill is also reflected in early reading as you put letters together to make a more complex word or sound. These skills appear to be closely related, and it is possible that early mathematics skill acquisition and early literacy skill acquisition may demonstrate correlations with one another even at this early level.

*Summary*

This study sought to investigate several early mathematics skills that have been demonstrated in the literature to be relevant in early childhood development. Overall, this research confirmed that children are able to demonstrate many more skills than are often measured as part of the educational process in school systems. Furthermore, a strong model of Early Childhood Mathematics Skill Acquisition was identified. This research adds more definition to the current literature on early mathematics skill acquisition, and directly relates to testing and designing instruction in order to target specific skill deficits as is now required by federal and state law.

## References

- Anthony, J.L., & Lonigan, C.J. (2004). The nature of phonological awareness: Converging evidence from four studies of preschool and early grade school children. *Journal of Educational Psychology, 96*, 43-55.
- Bertelli, R., Joanni, E., & Martlew, M. (1998). Relationship between children's counting ability and their ability to reason about number. *European Journal of Psychology Education, 13*, 371-384.
- Bowman, B. T., Donovan, M. S., & Burns, M. S. (Eds.). (2000). *Eager to learn: Educating our preschoolers*. Washington DC: National Academy Press.
- Bradley, R., Danielson, L., & Doolittle, J. (2005). Response to Intervention. *Journal of Learning Disabilities, 38*, 485 – 486.
- Briars, D., & Siegler, R. S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology, 20*, 607 – 618.
- Bruner, J. S. (1972). The nature and uses of immaturity. *American Psychologist, 27*, 687 - 708.
- Campbell, F.A., Pungello, E.P., Miller-Johnson, S., Burchinal, M., & Ramey, C.T. (2001). The development of cognitive and academic activities: Growth curves from an early childhood educational experiment. *Developmental Psychology, 37*, 231-242.
- Chaney, C. (1998). Preschool language and metalinguistic skills are links to reading success. *Applied Psycholinguistics, 19*, 433-446.
- Clarke, B., & Shinn, M. (2004). *Test of early numeracy (TEN): Administration and scoring of AIMSweb early numeracy measures for use with AIMSweb*. NCS

Pearson, Inc.

- Clarke, B., & Shinn, M. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review, 33*, 234 – 248.
- Coyne, M. D., & Harn, B. A. (2006). Promoting beginning reading success through meaningful assessment of early literacy skills. *Psychology in the Schools, 43*, 33 – 43.
- Deno, S. L. (1985). Curriculum-based measurement: The emerging alternative. *Exceptional Children, 52*, 219-232.
- Deno, S. L. & Fuchs, L. S. (1987). Developing curriculum-based measurement systems for data based special education problem solving. *Focus on Exceptional Children, 19*, 1- 15.
- Deno, S. L. & Mirkin, P. K. (1977). *Data-based program modification: A manual*. Reston, VA: Council for Exceptional Children.
- Dickinson, D.K., McCabe, A., Anastasopoulos, L., Peisner-Feinberg, E.S., & Poe, M.D. (2003). The comprehensive language approach to early literacy: The interrelationships among vocabulary, phonological sensitivity, and print knowledge among preschool-aged children. *Journal of Educational Psychology, 95*, 465-481.
- Education Commission of the States (2002). No state left behind: The challenges and opportunities of ESEA 2001. *ECS Special Report*. Retrieved December 2, 2004 from <http://www.ecs.org/clearinghouse/32/37/3237.doc>
- Fuchs, D., Mock, D., Morgan, P. L., & Young, C. L. (2003). Responsiveness-to-

- intervention: Definitions, evidence, and implications for the learning disabilities construct. *Learning Disabilities Research & Practice, 18*, 157-171.
- Fuchs, L., & Fuchs, D. (2002). Mathematical problem-solving profiles of students with mathematics disabilities with and without comorbid reading disabilities. *Journal of Learning Disabilities, 35*, 563 – 574.
- Fuchs, L., Fuchs, D., & Prentice, K. (2004). Responsiveness to mathematical problem-solving instruction: Comparing students at risk of mathematics disability with and without risk of reading disability. *Journal of Learning Disabilities, 37*, 293-306.
- Gelman, R., & Gallistel, R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.
- Gelman, R., & Meck, E. (1983). Preschoolers' counting: Principles before skill. *Cognition, 13*, 343-359.
- Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38*, 293 – 304.
- Ginsburg, H.P., & Baroody, A.J. (2003). *Test of Early Mathematics Ability, Third Edition*. Los Angeles CA: Western Psychological Services.
- Ginsburg, H. P., Inoue, N., & Seo, K. (1999). Young children doing mathematics. (Ch 9). In J. Copley, Ed., *Mathematics in the Early Years*, National Association for the Education of Young Children, Washington DC.
- Ginsburg, H. P., & Seo, K. (2000). Preschoolers' mathematical reading. *Teaching Children Mathematics, 7*, 226 – 229.
- Good, R.H., & Kaminski, R.A. (1996). Assessment for instructional decision-making for

- early literacy skills. *School Psychology Quarterly*, *11*, 326-336.
- Group Mathematics Assessment and Diagnostic Evaluation (2008). Pearson Education Inc.
- Henniger, M. L. (1987). Learning mathematics and science through play. *Childhood Education*, *63*, 167 – 171.
- Howell, D. C. (2002). *Statistical methods for psychology*. Pacific Grove, CA: Thompson Learning, Inc.
- Hu, L., & Bentler, P.M. (1999). Cutoff criteria for fit indices in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling*, *6*, 1 – 55.
- Individuals with Disabilities Education Act of 2004, 34 CFR § 300 (2006).
- Klein, J. S., & Bisanz, J. (2000). Preschoolers doing arithmetic: The concepts are willing but the working memory is weak. *Canadian Journal of Experimental Psychology*, *54*, 105-116.
- Kulak, A. (1993). Parallels between math and reading disability: Common issues and approaches. *Journal of Learning Disabilities*, *26*, 666-674.
- Light, J.G., & DeFries, J.C. (1995). Comorbidity of reading and mathematics disabilities: Genetic and environmental etiologies. *Journal of Learning Disabilities*, *28*, 96 – 107.
- Locuniak, M. N., & Jordan, N. C. (2008). Using kindergarten number sense to predict calculation fluency in second grade. *Journal of Learning Disabilities*, *41*, 451-459.
- Lonigan, C.J., Burgess, S.R., & Anthony, J.L. (2000). Development of emergent literacy

- and early reading skills in preschool children: Evidence from a latent-variable longitudinal study. *Developmental Psychology*, 36, 596-613.
- Lonigan, C.J., Burgess, S.R., Anthony, J.L., & Barker, T.A. (1998). Development of phonological sensitivity in two-to-five-year-old children. *Journal of Educational Psychology*, 90, 294-311.
- MacLean, M., Bryant, P., & Bradley, L. (1987). Rhymes, nursery rhymes, and reading in early childhood. *Merrill-Palmer Quarterly*, 33, 255-281.
- Murray, P. L., & Mayer, R. E. (1988). Preschool children's judgments of number magnitude. *Journal of Educational Psychology*, 80, 206 - 209.
- National Mathematics Advisory Panel (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*, U.S. Department of Education: Washington DC.
- Normandeau, S., & Guay, F. (1998). Preschool behavior and first-grade achievement: The mediational role of cognitive self-control. *Journal of Educational Psychology*, 90, 111-121.
- Pearson (2009). *Wechsler Individual Achievement Test, Third Edition*. San Antonio, TX: Pearson Education, Inc.
- Rasanen, P., & Ahonen, T. (1995). Arithmetic disabilities with and without reading difficulties: A comparison of arithmetic errors. *Developmental Neuropsychology*, 11, 275 – 295.
- Regulations of the New York State Commissioner of Education Relating to General Education and Diploma Requirements, 8 NYCRR § 100 (2007).
- Regulations of the New York State Commissioner of Education Relating to Students with

- Disabilities, 8 NYCRR § 200 (2007).
- Resnick, L.B. (1989). Developing mathematical knowledge. *American Psychologist*, 44, 162-169.
- Shinn, M. R. (Ed.) (1989). *Curriculum-based measurement: Assessing special children*. New York: Guilford.
- Sophian, C. (2000). From objects to quantities: Developments in preschool children's judgments about aggregate amount. *Developmental Psychology*, 36, 724 – 730.
- Spellings, M. (2007). Building on results: A blueprint for strengthening the No Child Left Behind Act. *U.S. Department of Education*.
- Stahl, S.A., & Murray, B.A. (1994). Defining phonological awareness and its relationship to early reading. *Journal of Educational Psychology*, 86, 221-234.
- Stannard, L., Wolfgang, C. H., Jones, I., & Phelps, P. (2001). A longitudinal study of the predictive relations among construction play and mathematical achievement. *Early Child Development and Care*, 167, 115 – 125.
- Stansfield, W.D. (2002). Occam's razor and the nature of scientific theories. *The American Biology Teacher*, 64, 107-110.
- Stevenson, H.W., & Newman, R.S. (1986). Long-term prediction of achievement and attitudes in mathematics and reading. *Child Development*, 57, 646-659.
- Tunmer, W.E., Herriman, M.L., & Nesdale, A.R. (1988). Metalinguistic abilities and beginning reading. *Reading Research Quarterly*, 23, 134-158.
- University of Oregon Center on Teaching and Learning (n.d.). *DIBELS data system*. Retrieved November 8, 2008, from <https://dibels.uoregon.edu/>.

- VanDerHeyden, A. M., Witt, J. C., Gilbertson, D. (2007). A multi-year evaluation of the effects of a Response to Intervention (RTI) model on identification of children for special education. *Journal of School Psychology, 45*, 225 – 256.
- Vellutino, F. R., Scanlon, D. M., Small, S., & Fanuele, D. P. (2006). Response to Intervention as a vehicle for distinguishing between children with and without reading disabilities: Evidence for the role of kindergarten and first-grade interventions. *Journal of Learning Disabilities, 39*, 157-169.
- Wagner, R. K., & Torgesen, J. K. (1997). The nature of phonological processing and its causal role in the acquisition of reading skills. *Psychological Bulletin, 101*, 192 – 212.
- Whitehurst, G.J., & Lonigan, C.J. (1998). Child development and emergent literacy. *Child Development, 69*, 848-872.
- Whitehurst, G.J., & Lonigan, C.J. (2001). Emergent literacy: Development from pre readers to readers. In S.B. Neuman & D.K. Dickinson (Eds.), *Handbook of early literacy research* (pp.11-29). New York: Guilford Press.
- Wolfgang, C. H., Stannard, L. L., & Jones, I. (2001). Block play performance among preschoolers as a predictor of later school achievement in mathematics. *Journal of Research in Childhood Education, 15*, 173 – 180.
- Wright, J. (2008). *Early math fluency CBM probe: Number Identification*. Retrieved March 31, 2008, from [http://www.jimwrightonline.com/pdfdocs/numberfly/NID\\_directions.pdf](http://www.jimwrightonline.com/pdfdocs/numberfly/NID_directions.pdf)

Table 1

*Descriptive Statistics for Demographic Variables*

	Mean	Standard Deviation	Range
Age*	61.60	23.00	3.80
Years in Preschool	1.30	0.80	4.50
SES (Mother's Education)**	5.48	1.80	8.00

\*Age is measured in months

\*\* SES was measured by Mother's level of education. Coding for this was 1 = Some High School; 2 = GED; 3 = High School Graduate; 4 = Some College; 5 = Associate's Degree; 6 = Bachelor's Degree; 7 = Some Graduate School; 8 = Master's Degree; 9 = Doctoral Degree

Table 2

*Descriptive Statistics for Early Mathematics Measures*

	Mean	Standard Deviation	Range
One to One Correspondence Principle	3.25	0.91	4.00
Stable Order Principle	3.53	0.74	2.00
Cardinality Principle	3.35	0.80	3.00
Order Irrelevance Principle	4.07	1.22	6.00
Number Identification	3.64	2.00	7.00
Oral Counting	3.82	2.12	8.50
Addition and Subtraction	5.93	1.73	8.00
One to One Correspondence Task (Lady Bug)	2.74	0.51	2.00
Magnitude	6.57	1.67	8.00

Figure 1: Hypothesized Model

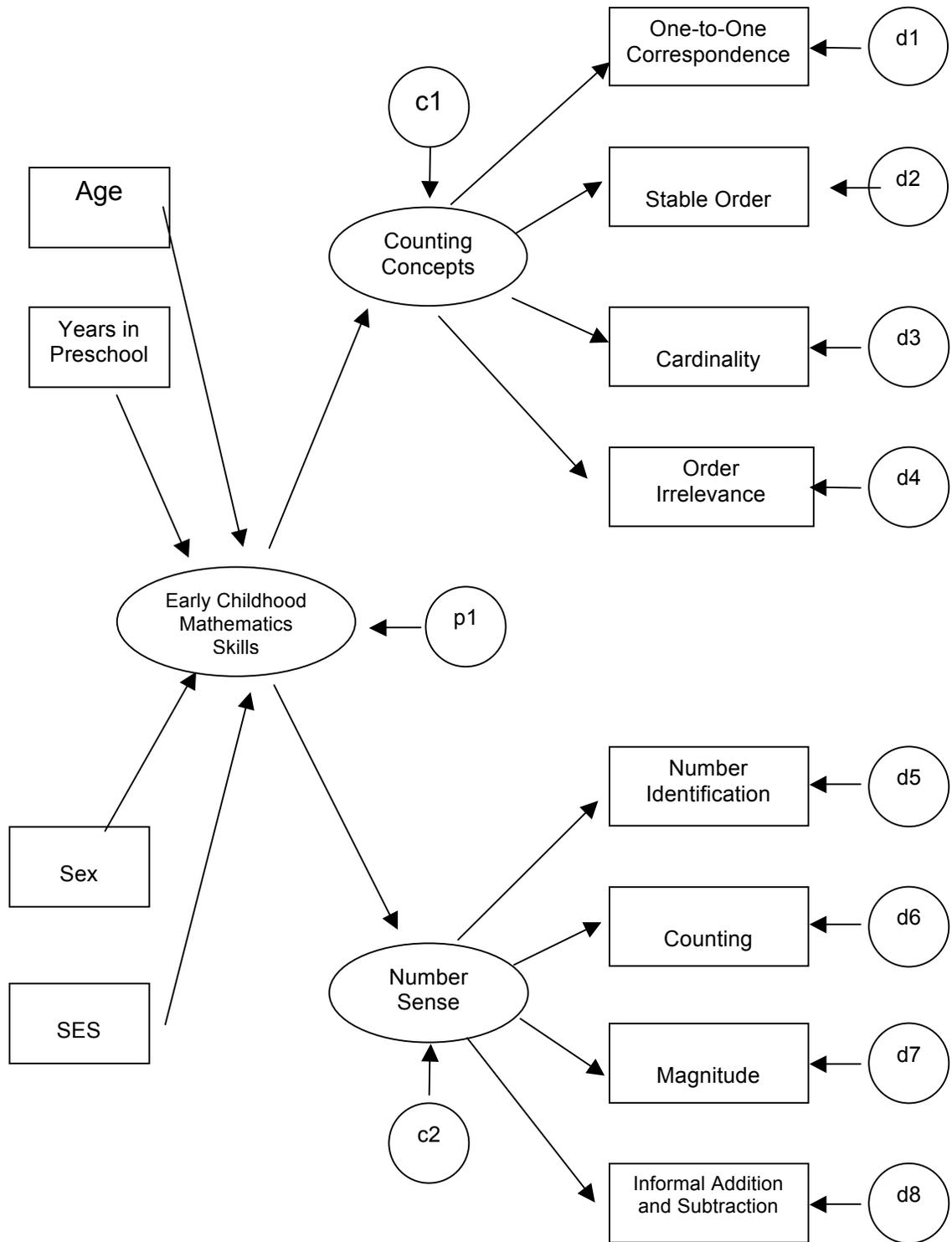


Figure 2: Counting Concepts Confirmatory Factor Analysis

Chi Square = 8.783  
 DF = 15  
 $p = .889$   
 RMSEA = .000  
 CFI = 1.0

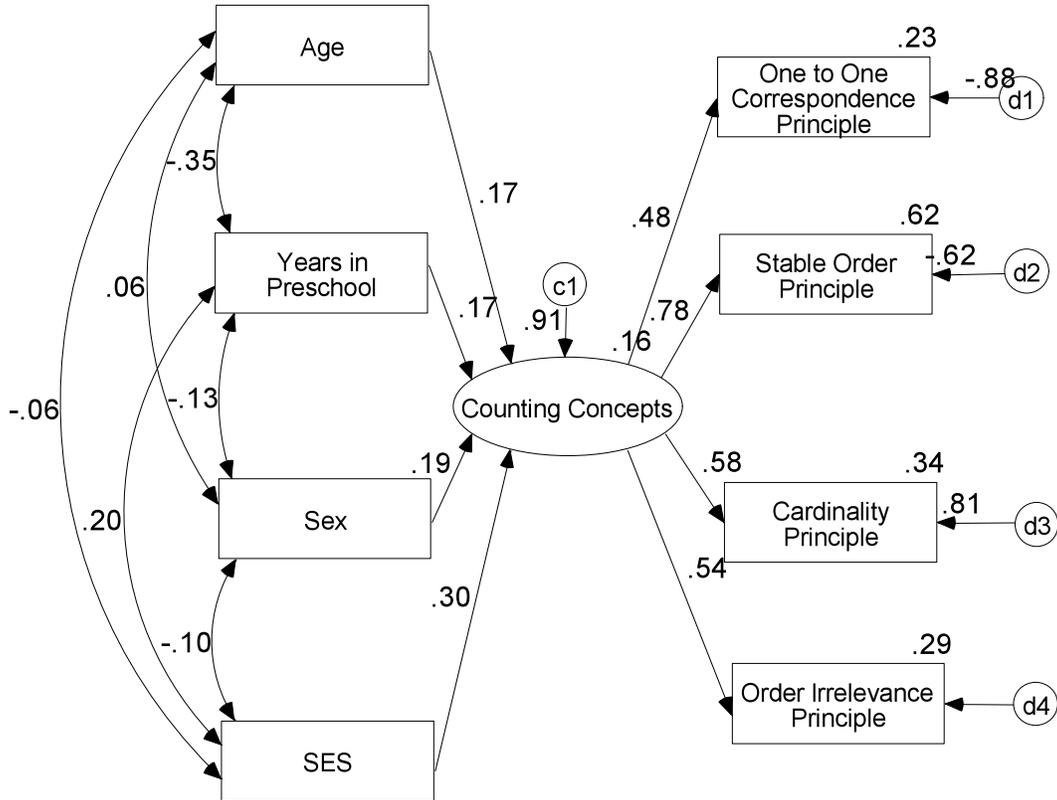


Figure 3: Original Number Sense Confirmatory Factor Analysis

Chi Square = 30.403  
 DF = 15  
 $p < .001$   
 RMSEA = .109  
 CFI = .735

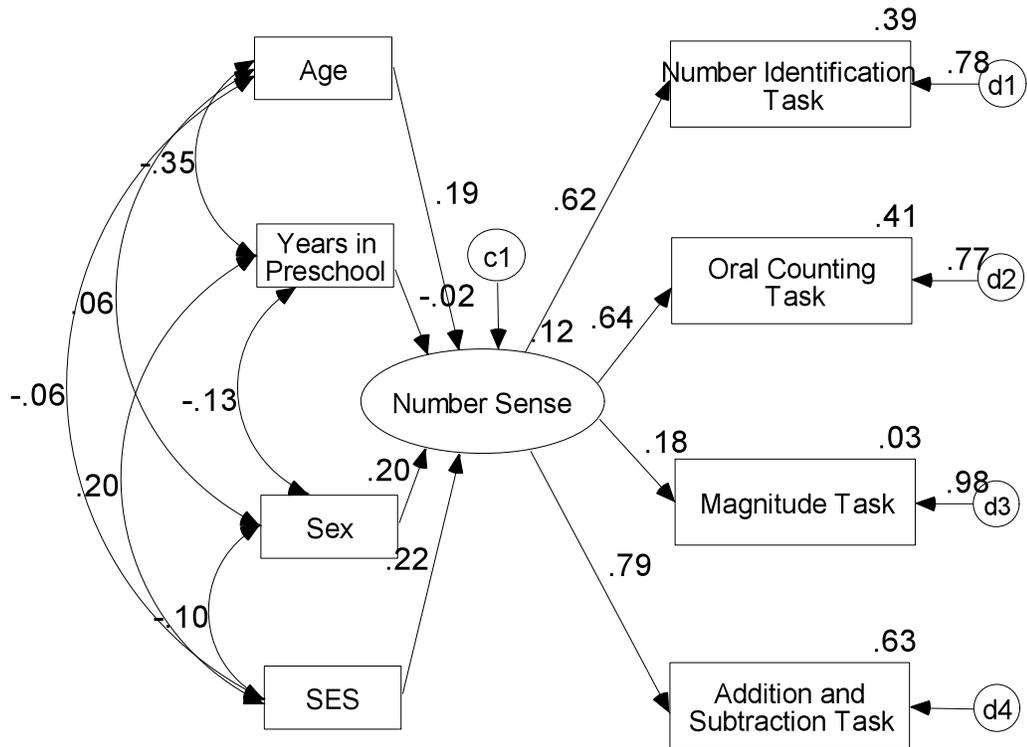


Figure 4: Final Number Sense Confirmatory Factor Analysis

Chi Square = 22.081  
 DF = 21  
 $p = .395$   
 RMSEA = .024  
 CFI = .986

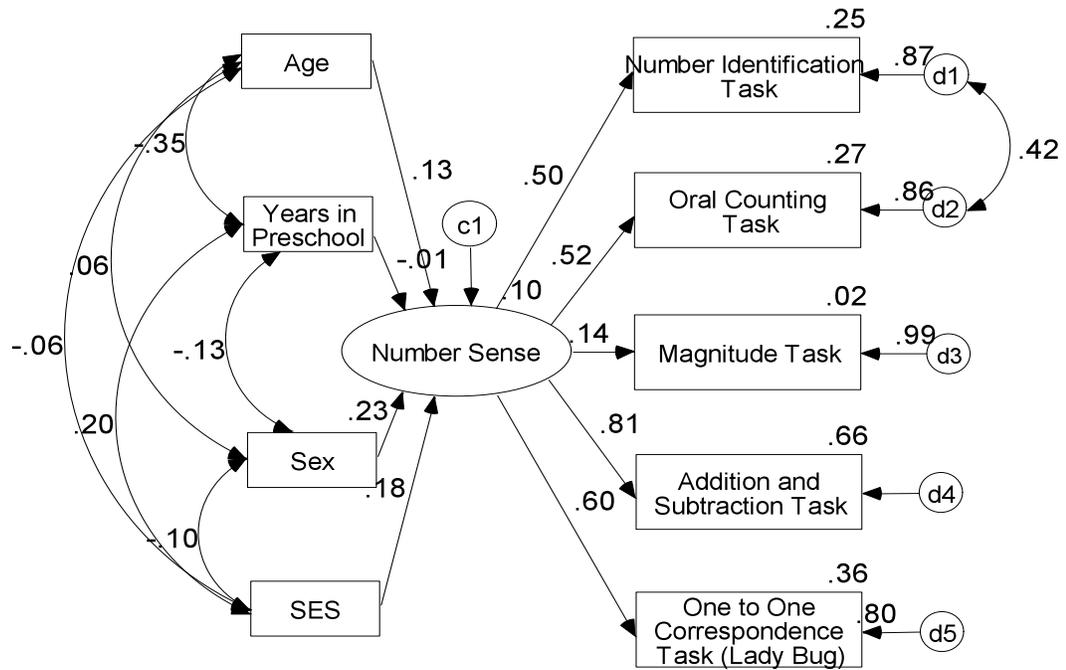


Figure 5: Hypothesized Model Confirmatory Factor Analysis

Chi Square =  
 135.360  
 DF = 65  
 $p < .001$   
 RMSEA = .112

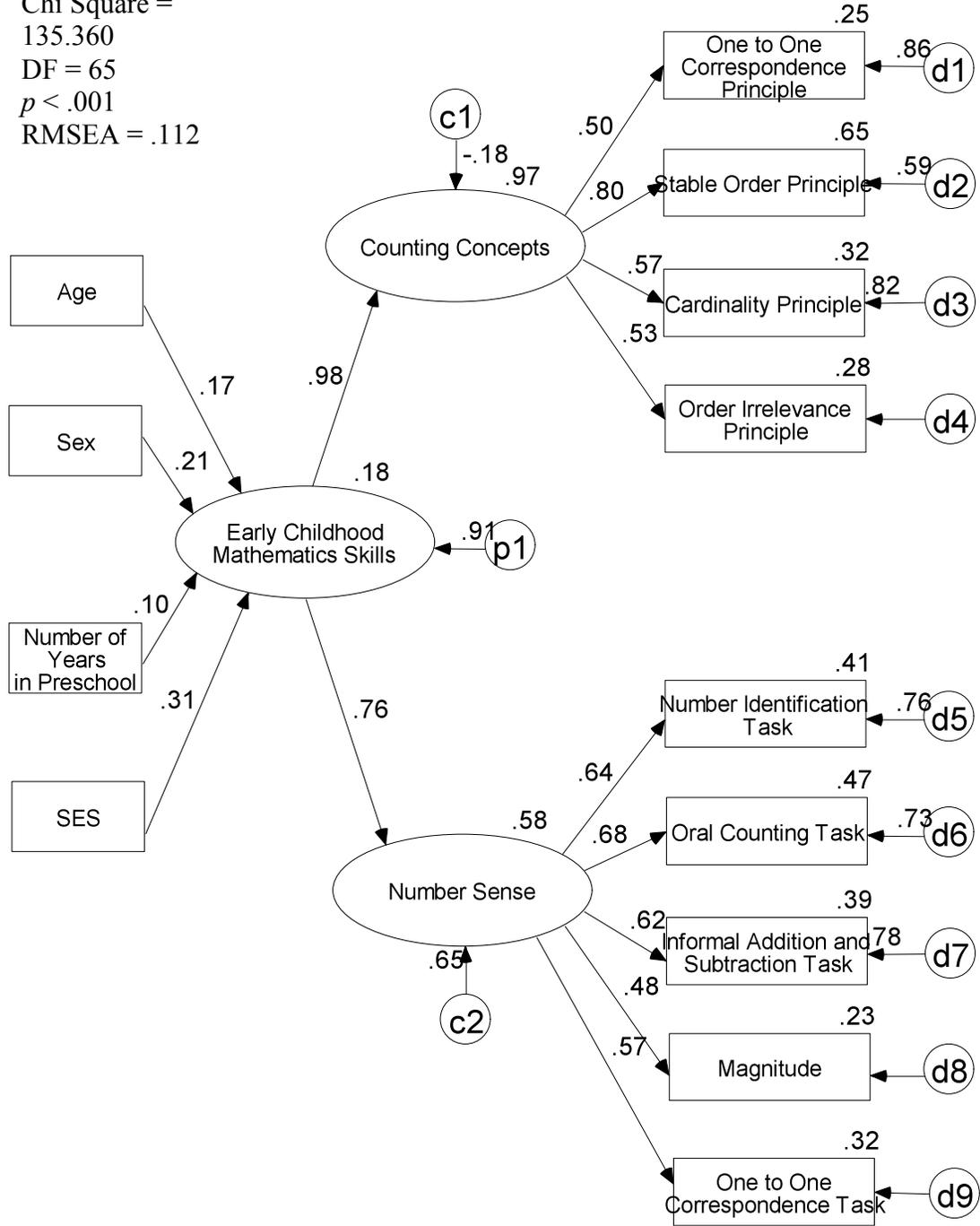
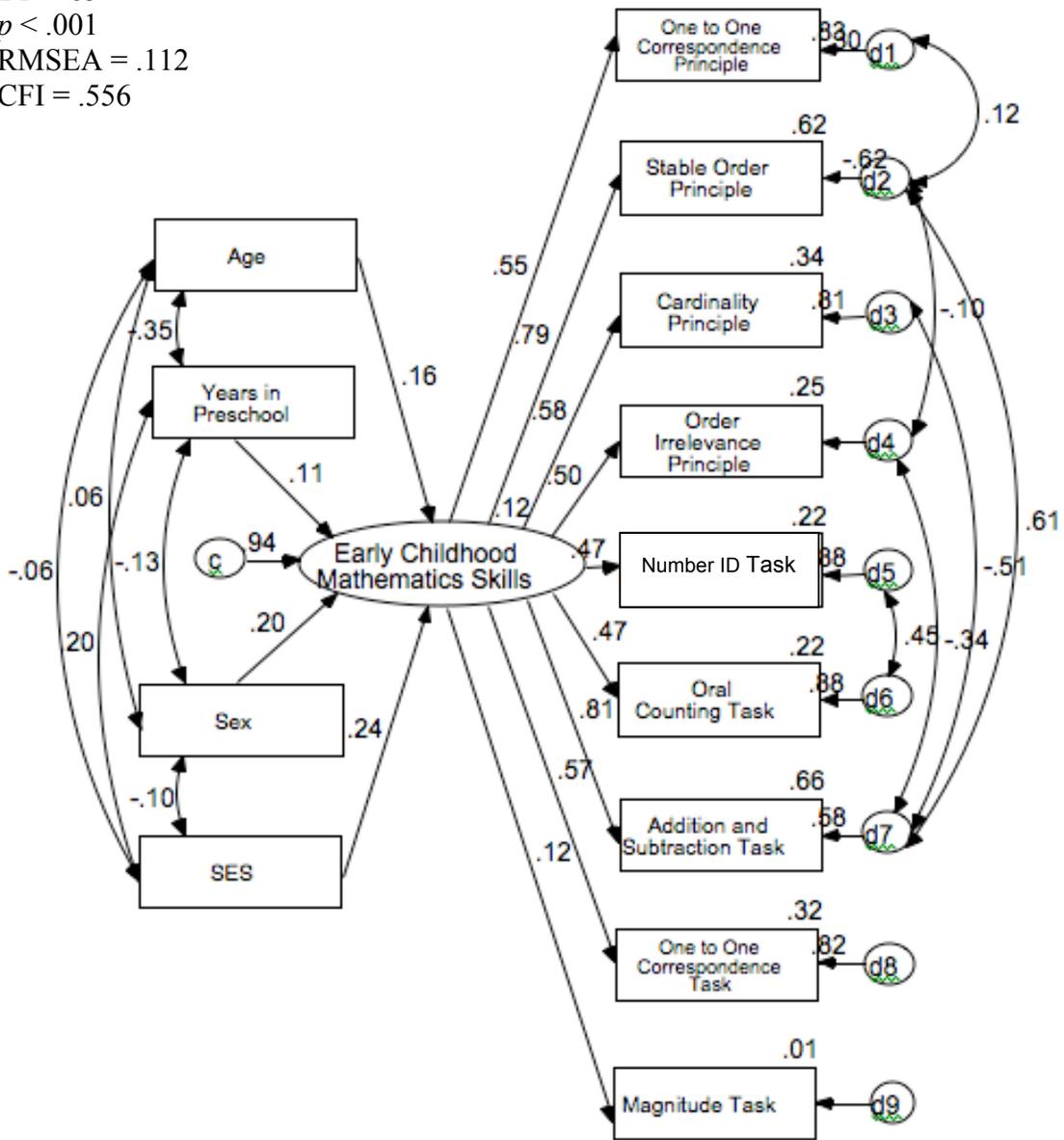


Figure 6: Final Model Confirmatory Factor Analysis

Chi Square = 135.360  
 DF = 65  
 $p < .001$   
 RMSEA = .112  
 CFI = .556



Appendix A: Informed Consent

**Cara A. Smith**  
**Alfred University**  
**Graduate School, Division of School Psychology**

Dear Parent(s) or Guardian(s):

I am a graduate student under the direction of Dr. Mark Fugate in the Department of School Psychology at Alfred University. I am conducting a research study to examine mathematics skills in preschool students in order to identify an appropriate model for early mathematics instruction.

Your child's participation will involve the completion of eight very short and simple mathematics tasks in a one-on-one situation with a trained researcher. It is likely that this participation will last between 15 and 20 minutes. Most children truly enjoy this type of one-on-one attention. However, if your child demonstrates any unusual signs of stress, he or she will be returned to their classroom without needing to complete any more tasks. In addition, your participation, as well as that of your child, in this study is voluntary. If you or your child choose not to participate or to withdraw from the study at any time, there will be no penalty. It will not affect your child's grade, treatment, or care. The results of the research study may be published, but your child's name will not be used and all information obtained from you and your child will be treated with confidentiality. This research has been approved by Alfred University's Institutional Review Board.

Although there may be no direct benefit to your child, the possible benefit of your child's participation is that their teachers may begin to focus on different areas of math in order to enhance your child's or other students' later learning in mathematics.

If you have any questions concerning this research study or your child's participation in the study, please call me at (607) 758-4155 or email me at [caf2@alfred.edu](mailto:caf2@alfred.edu). You may also reach my advisor, Dr. Mark Fugate at (607) 871-2212 or [ffugate@alfred.edu](mailto:ffugate@alfred.edu). Please complete the following consent and informational form.

Sincerely,

Cara A. Smith, MA CAS  
Alfred University  
1 Saxon Drive  
Alfred, NY 14802

If you have any questions about your rights as a participant in this research, you can contact Jana Atlas, Ph.D., chairperson of Alfred University's Human Subjects Research Committee, at (607) 871-2212 or [atlasj@alfred.edu](mailto:atlasj@alfred.edu).

**Cara A. Smith  
Alfred University  
Graduate School, Division of School Psychology**

**Parental Consent form:**

I give consent for my child \_\_\_\_\_ to participate in the study on early mathematics.

Parent's Name (print): \_\_\_\_\_

Parent's Signature \_\_\_\_\_ (Date) \_\_\_\_\_

-----

**Student Name:** \_\_\_\_\_ **DOB:** \_\_\_\_\_

**Gender:** (Please Circle 1)    Male    Female

**Current School:** \_\_\_\_\_

**Length of time child has been in preschool:** \_\_\_\_\_

**Mother's Level of Education:** \_\_\_\_\_

**Contact Information to set up appointment:**

**Teacher:** \_\_\_\_\_

If you would like to receive a summary of results of the completed study, please check the box below.

Please send me a copy of a summary of findings from your study on preschool mathematics skills.

Appendix B: One-to-One Correspondence - Modified from Gelman & Meck, 1983

Materials:

- Puppet (Mr. Lion)
- Bag of 20 blocks – 10 red, 10 blue

1. Set out 3 blue and 3 red blocks, alternating in color.

***“Look at these blocks.”*** Take out Lion puppet. ***“This is my friend, Mr. Lion. He would like you to help him play a game. Mr. Lion is going to count the blocks on the table but Mr. Lion is just learning how to count and sometimes makes mistakes. Sometimes he counts in ways that are OK but sometimes he counts in ways that are not OK and that are wrong. It is your job to tell him after he finishes counting if it was OK to count the way that he did or not OK. So remember, you have to tell him if he counts in a way that is OK or in a way that is not OK and wrong.”***

2. The lion counts the row of 6 blocks, pointing to the blocks as he counts them:

					
1	2	3	4	5	6

Ask child, ***“Did Mr. Lion count in a way that is OK or in a way that is not OK and wrong?”***

OK                      Not OK

3. The lion counts the same row of blocks, pointing to the blocks that he counts.

Say to child, ***“Mr. Lion is going to try again. Remember to tell him if he counted OK or not OK and wrong.”***

					
1	2	Skip	3	4	5

Ask child, ***“Did Mr. Lion count in a way that is OK or in a way that is not OK and wrong?”***

OK                      Not OK

4. Put all of the blocks in a bag and then put out an array of 12 alternating red and blue blocks.

Say to child, ***“Now Mr. Lion is going to try to count more blocks.”***

											
1	2	3	4	5	6	7,8	9	10	11	12	13

Ask child, ***“Did Mr. Lion count in a way that is OK or in a way that is not OK and wrong?”***

OK                      Not OK

5. The lion counts the same row of blocks, pointing to the blocks that he counts.

Say to child, ***“Mr. Lion is going to try again. Remember to tell him if he counted OK or not OK and wrong.”***

                                             
1    2    3    4    5    6    7    8    9    10    11    12

Ask child, ***“Did Mr. Lion count in a way that is OK or in a way that is not OK and wrong?”***

OK                      Not OK

6. Clear blocks.

Appendix C: Stable Order Principle - Modified from Gelman & Meck, 1983

Materials:

- Puppet (Mr. Lion)
- Bag of 20 objects

1. Say to child, ***“Now Mr. Lion is going to count some more things.”***

Set out 5 objects.

X	X	X	X	X
<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>

***“Did Mr. Lion count right?”***

Yes      No

2. Put 5 objects into the bag and set up 12 objects.

X	X	X	X	X	X	X	X	X	X	X	X
<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>	<i>11</i>	<i>12</i>

***“Did Mr. Lion count right?”***

Yes      No

3. Put 12 objects into the bag and set up 7 objects.

X	X	X	X	X	X	X
<i>1</i>	<i>2</i>	<i>4</i>	<i>3</i>	<i>5</i>	<i>6</i>	<i>7</i>

***“Did Mr. Lion count right?”***

Yes      No

4. Put 7 objects into the bag and set up 20 objects.

X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>	<i>11</i>	<i>12</i>	<i>13</i>	<i>14</i>	<i>15</i>	<i>16</i>	<i>17</i>	<i>18</i>	<i>19</i>	<i>20</i>	<i>21</i>

***“Did Mr. Lion count right?”***

Yes      No

5. Clear objects.

Appendix D: Cardinality - Modified from Gelman & Meck, 1983

Materials:

- Puppet (Mr. Lion)
- Bag of 20 objects

1. Say to child, "***Now Mr. Lion is going to count the objects and tell us how many there are. Your job is to tell him if he is right or if he is wrong.***"
2. Set up 5 objects in a row. "***Let's watch Mr. Lion count.***"

X	X	X	X	X
<b><i>1</i></b>	<b><i>2</i></b>	<b><i>3</i></b>	<b><i>4</i></b>	<b><i>5</i></b>

Examiner to puppet: "***How many?***"

Mr. Lion: "***5***"

Examiner looks at the child and asks, "***Is he right?***"

Yes                      No

3. Clear previous objects and set up 7 objects in a row. "***Let's watch Mr. Lion count these.***"

X	X	X	X	X	X	X
<b><i>1</i></b>	<b><i>2</i></b>	<b><i>3</i></b>	<b><i>4</i></b>	<b><i>5</i></b>	<b><i>6</i></b>	<b><i>7</i></b>

Examiner to puppet: "***How many?***"

Mr. Lion: "***10***"

Examiner looks at the child and asks, "***Is he right?***"

Yes                      No

4. Clear previous objects and set up 20 objects in a row. "***Mr. Lion is going to count again.***"

X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
<b><i>1</i></b>	<b><i>2</i></b>	<b><i>3</i></b>	<b><i>4</i></b>	<b><i>5</i></b>	<b><i>6</i></b>	<b><i>7</i></b>	<b><i>8</i></b>	<b><i>9</i></b>	<b><i>10</i></b>	<b><i>11</i></b>	<b><i>12</i></b>	<b><i>13</i></b>	<b><i>14</i></b>	<b><i>15</i></b>	<b><i>16</i></b>	<b><i>17</i></b>	<b><i>18</i></b>	<b><i>19</i></b>	<b><i>20</i></b>

Examiner to puppet: "***How many?***"

Mr. Lion: "***20***"

Examiner looks at the child and asks, ***“Is he right?”***

Yes                      No

5. Clear previous objects and set up 12 objects in a row. ***“Get ready, Mr. Lion is going to count.”***

X	X	X	X	X	X	X	X	X	X	X	X
<b><i>1</i></b>	<b><i>2</i></b>	<b><i>3</i></b>	<b><i>4</i></b>	<b><i>5</i></b>	<b><i>6</i></b>	<b><i>7</i></b>	<b><i>8</i></b>	<b><i>9</i></b>	<b><i>10</i></b>	<b><i>11</i></b>	<b><i>12</i></b>

Examiner to puppet: ***“How many?”***

Mr. Lion: ***“9”***

Examiner looks at the child and asks, ***“Is he right?”***

Yes                      No

6. Clear objects.

Appendix E: Order Irrelevance – Modified from Briars & Siegler, 1984

Materials:

- Puppet (Callie Cat)
- Cardboard with 10 green and yellow plastic chips pasted on it

***“Now I want you to meet Callie Cat. Callie Cat is also learning how to count. Sometimes when she counts she makes mistakes. She knows her numbers, but sometimes when she counts the chips, she does it wrong. Callie Cat is going to count for you. I want you to watch her carefully to see if she makes a mistake. Let’s try one for practice. Watch carefully.”***

Callie Cat counts 10 chips, skipping three adjacent chips in the middle of the row.

Ask the child, ***“Did Callie Cat make a mistake?”***

If the child says yes: ***“That’s right! Good job. Let’s try another one.”***

If the child says no: ***“That’s not quite right. Callie Cat missed a few in the middle. Let’s try that one again.”***

Repeat the same question and then go on to the experimental trials.

1. Count every green chip until reaching the end of the array and then return back counting the yellow chips in the opposite direction. Count correctly. (unusual)

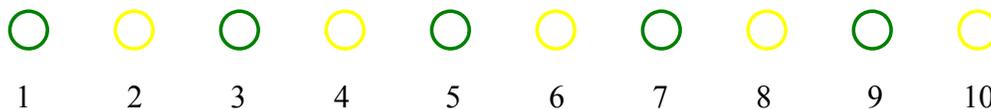


Examiner: ***“OK or Not OK?”***

OK Not OK

Examiner: ***“Let’s try another one.”***

2. Count all 10 chips correctly from left to right. (standard)



Examiner: ***“OK or Not OK?”***

OK Not OK

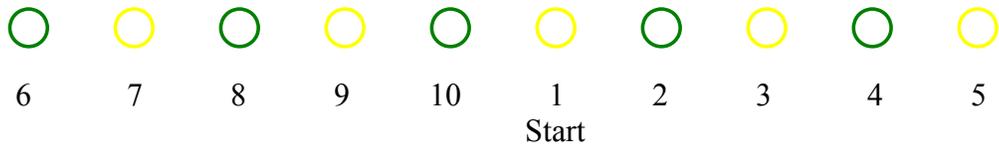
3. Count the chips omitting the 5<sup>th</sup> chip by pointing to it but not saying the word (error).



Examiner: ***“OK or Not OK?”***

OK                      Not OK

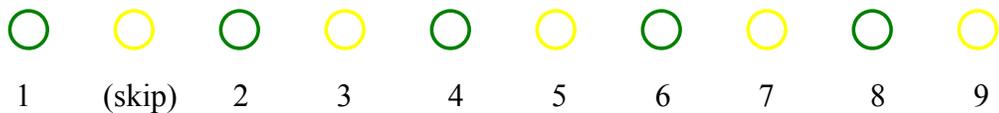
4. Count chips correctly, starting in the middle counting right to the end, and then starting over at the beginning to finish where started. (unusual).



Examiner: ***“OK or Not OK?”***

OK                      Not OK

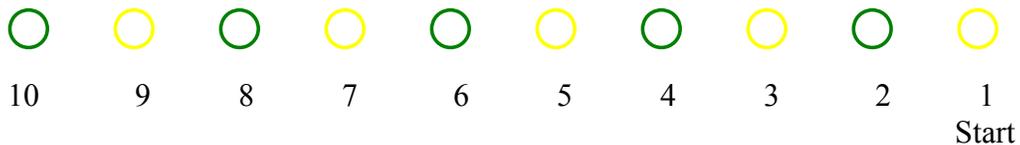
5. Count the chips by skipping one object altogether (error).



Examiner: ***“OK or Not OK?”***

OK                      Not OK

6. Count all chips correctly, from right to left (unusual).



Examiner: ***“OK or Not OK?”***

OK                      Not OK

7. Violate word/object correspondence by assigning too many number words to one object (error).

1     2     3,4     5     6     7     8     9     10     11

Examiner: ***“OK or Not OK?”***

OK

Not OK

Appendix F: Number Identification – Clarke & Shinn, 2004

1. The examiner sits with the student in a quiet area without distractions. The examiner sits at a table across from the student.
2. Examiner: ***“Look at the paper in front of you. It has a number on it (demonstrate by pointing). What number is this.”***

Example 1:

<p><b>CORRECT RESPONSE:</b>                  “Good. The number is 8. Look at the number next to 8 (demonstrate by pointing). What number is this?”</p>	<p><b>INCORRECT RESPONSE</b>                  “This number is 8 (point to 8). What number is this? Good. Let’s try another one. Look at the number next to 8 (demonstrate by pointing). What number is this?”</p>
--	---

Example 2:

<p><b>CORRECT RESPONSE:</b>                  “Good. The number is 4.” (Turn the page).</p>	<p><b>INCORRECT RESPONSE</b>                  “This number is 4 (point to 4). What number is this? Good. (Turn the page).</p>
--	---

***“The paper in front of you has numbers on it. When I say start, I want you to tell me what the numbers are. Start here and go across the page (demonstrate by pointing). If you come to a number you don’t know, I’ll tell you what to do. Are there any questions? Put your finger on the first one. Ready, start.”***

3. The examiner begins the stopwatch. If the student hesitates on a number for 3 seconds or longer, the examiner says, *“Try the next one.”*
4. The examiner marks each Number Identification error by marking a slash (/) through the incorrectly read number on the examiner form.
5. At the end of one minute, the examiner says, ***“Stop”*** and writes a right-bracket symbol on the examiner form from the point in the number series that the student had reached when the time expired. The examiner then collects the student Number Identification sheet.
6. To score:

Correct NID responses include:

- Numbers read correctly
- Numbers read incorrectly but corrected by the student within 3 seconds

Incorrect NID responses include:

- Numbers read incorrectly
- Numbers read correctly after hesitations of 3 seconds or longer

- Numbers skipped by the student
- Entire rows of numbers skipped by a student

To calculate a Number Identification fluency score, the examiner:

- 1) counts up all numbers that the student attempted to read aloud and
- 2) subtracts the number of errors from the total of numbers attempted.
- 3) The resulting figure is the number of correct numbers identified.(NID fluency score).

## Appendix G: Counting Task – Oral Counting by Clarke & Shinn (2004)

### Materials:

- Oral Counting protocol

### AIMSweb Oral Counting

1. Place the examiner copy on a clipboard and position so the student cannot see what the examiner records.
2. Say these specific directions to the student:  
  
*“When I say start I want you to start counting aloud from 1 like this 1, 2, 3 until I tell you to stop. If you come to a number you don’t know I’ll tell it to you. Be sure to do your best counting. Are there any questions? Ready, start.”*
3. Start your stopwatch. If the student fails to say “1” after 3 seconds, say “1” and continue.
4. Follow along on the examiner copy. Score according to scoring rules. After one minute has expired, place a bracket after the last number said and say **“Stop.”**
5. To score:

#### Correct responses include:

- Numbers stated correctly
- Numbers stated incorrectly but corrected by the student within 3 seconds

#### Incorrect Oral Counting responses include:

- Skipped numbers while counting (i.e., 6, 7, 9)
- Hesitations of 3 seconds or longer

#### To calculate a Oral Counting score, the examiner:

- 1) Counts up all numbers that the student said
- 2) Subtracts the number of errors from the total of numbers attempted.
- 3) The resulting figure is the number correct

## Appendix H: Lady Bug Counting

Materials:

- 3 Ladybug pictures (In binder)

Hand child 1 ladybug at a time: 7 dots, 3 dots, 5 dots

**1. “How many dots does this ladybug have on her wings? Point to each dot and count them out loud for me.”**

7 dots \_\_\_\_\_ # Correct \_\_\_\_\_

3 dots \_\_\_\_\_ # Correct \_\_\_\_\_

5 dots \_\_\_\_\_ # Correct \_\_\_\_\_

Total Correct \_\_\_\_\_

Appendix I: Magnitude – Modified from Sophian, 2000

Materials:

- Puppet
- 1 block, 2 dimes
- Pages of “cookies” (in binder)

Say to Child: ***“Remember Callie Cat? All of that counting sure made him hungry. Now he wants to fill his belly with cookies. He doesn’t care how many cookies he gets, so long as he gets lots to eat. Help Callie Cat decide which set of cookies Callie Cat should eat so that he gets as much as he can.”***

Introduction:

Show child one block and two dimes. Say to the child, ***“Let’s pretend that these are cookies. If Callie Cat can choose between eating this cookie (point to block) or these two cookies (point to two dimes), which one would fill up his belly more?”***

If child responds correctly, go on to experimental trials. If child responds incorrectly, say, ***“Well, let’s take a look. This cookie (point to block) is much larger than these two smaller cookies (point to two dimes). So, if Callie Cat ate this cookie, he would be all full. Let’s try that one again.”*** Repeat introduction item.

1. Say to child, ***“Which cookies would fill up Callie Cat’s belly the most? This one (point to first page) or this one (point to second page)?***  
(2 green - conflict) Correct \_\_\_\_\_ Incorrect \_\_\_\_\_
2. ***“Let’s try another one. Which cookies would fill up Callie Cat’s belly the most? This one (point to first page) or this one (point to second page)?***  
(5 magenta - agreement) Correct \_\_\_\_\_ Incorrect \_\_\_\_\_
3. ***“Let’s try another one. Which cookies would fill up Callie Cat’s belly the most? This one (point to first page) or this one (point to second page)?***  
(4 magenta - agreement) Correct \_\_\_\_\_ Incorrect \_\_\_\_\_
4. ***“Which cookies would fill up Callie Cat’s belly the most? This one (point to first page) or this one (point to second page)?***  
(4 magenta - agreement) Correct \_\_\_\_\_ Incorrect \_\_\_\_\_
5. ***“Which cookies would fill up Callie Cat’s belly the most? This one (point to first page) or this one (point to second page)?***  
(1 green - conflict) Correct \_\_\_\_\_ Incorrect \_\_\_\_\_
6. ***“Which cookies would fill up Callie Cat’s belly the most? This one (point to first page) or this one (point to second page)?***  
(5 magenta - agreement) Correct \_\_\_\_\_ Incorrect \_\_\_\_\_

7. ***“Which cookies would fill up Callie Cat’s belly the most? This one*** (point to first page) ***or this one*** (point to second page)?

(1 green - conflict)                      Correct \_\_\_\_\_ Incorrect \_\_\_\_\_

8. ***“Which cookies would fill up Callie Cat’s belly the most? This one*** (point to first page) ***or this one*** (point to second page)?

(2 green - conflict)                      Correct \_\_\_\_\_ Incorrect \_\_\_\_\_

Total # Conflict Correct: \_\_\_\_\_

Total # Agreement Correct: \_\_\_\_\_

Total # Correct: \_\_\_\_\_

Sizes of Cookies:

Amount Agreement:

1. 4 magenta circles = Area = 3.13  
2 green circles = Area = 2.5  
Ratio = 2.5:1
2. 4 magenta circles = Area = 5  
2 green circles = Area = 2.5  
Ratio = 4:1
3. 5 magenta circles = Area = 2.5  
1 green circle = Area = 5  
Ratio = 2.5:1
4. 5 magenta circles = Area = 4  
1 green circle = Area = 5  
Ratio = 4:1

Amount Conflict:

1. 4 magenta circles = Area = 1.25  
2 green circles = Area = 6.25  
Ratio = 2.5:1
2. 4 magenta circles = Area = 1.25  
2 green circles = Area = 10  
Ratio = 4:1
3. 5 magenta circles = Area = 1  
1 green circle = Area = 12.5  
Ratio = 2.5:1
4. 5 magenta circles = Area = 1  
1 green circle = Area = 20  
Ratio = 4:1

Appendix J: Informal Addition and Subtraction – Modified from Klein & Bisanz, 2000

Materials:

- Box
- Chips

Say to child, ***“Now we’re going to play a counting game.”***

Introduction: ***“Watch me.”*** Take one chip and place it on the table in front of you. Pause for 2 seconds and cover it with the box. Take another chip and place it under the box, without lifting the front of the box for the child to see.

Examiner: ***“Let’s see if you can put the same number of chips in front of you as are under the box.”***

If child answers correctly, continue with experimental trials. If child answers incorrectly, say, ***“That’s not quite right. Let’s try it again. Watch me carefully.”*** Repeat introduction.

1. ***“Let’s try some more. Remember to watch me carefully.”***

Place 2 chips on the table. Pause for 2 seconds and cover with the box. Add 1 more chip.

2 + 1            Correct \_\_\_\_\_            Incorrect \_\_\_\_\_

2. ***“Let’s try another one. Remember to watch me carefully.”***

Place 3 chips on the table. Pause for 2 seconds and cover with the box. Add 1 more chip.

3 + 1            Correct \_\_\_\_\_            Incorrect \_\_\_\_\_

3. ***“Ready? Watch me.”***

Place 1 chip on the table. Pause for 2 seconds and cover with the box. Add 2 more chips.

1 + 2            Correct \_\_\_\_\_            Incorrect \_\_\_\_\_

4. ***“Watch me.”***

Place 3 chips on the table. Pause for 2 seconds and cover with the box. Add 2 more chips.

3 + 2            Correct \_\_\_\_\_            Incorrect \_\_\_\_\_

Introduction subtraction: ***“Now we’re going to try it another way. Watch me.”***

Place 2 chips on the table. Pause for 2 seconds and cover with the box. Remove 1 chip.  
***“Show me how many chips are under the box.”***

If child answers correctly, continue with experimental trials. If child answers incorrectly, say, ***“That’s not quite right. Let’s try it again. Watch me carefully.”*** Repeat introduction.

5. ***“Let’s try some more. Remember to watch me carefully.”***

Place 4 chips on the table. Pause for 2 seconds and cover with the box. Subtract 1 chip and place it in the pile of chips on the table.

4 - 1            Correct \_\_\_\_\_            Incorrect \_\_\_\_\_

6. ***“Let’s try another one. Remember to watch me carefully.”***

Place 3 chips on the table. Pause for 2 seconds and cover with the box. Subtract 2 chips and place it in the pile of chips on the table.

3 - 2            Correct \_\_\_\_\_            Incorrect \_\_\_\_\_

7. ***“Ready? Watch me.”***

Place 3 chips on the table. Pause for 2 seconds and cover with the box. Subtract 1 chip and add it to the pile of chips on the table.

3 - 1            Correct \_\_\_\_\_            Incorrect \_\_\_\_\_

8. ***“Watch me.”***

Place 5 chips on the table. Pause for 2 seconds and cover with the box. Subtract 1 chip and add it to the pile of chips on the table.

5 - 1            Correct \_\_\_\_\_            Incorrect \_\_\_\_\_

Total # Addition correct: \_\_\_\_\_

Total # Subtraction correct: \_\_\_\_\_

Total # Correct: \_\_\_\_\_

## Appendix K: Review of the Relevant Literature

The current trend in early childhood education includes developing instructional methods that account for both general developmental abilities and unique child characteristics. At the preschool level, children tend to develop academic abilities at various rates when compared to their same-aged peers (Bowman, Donovan, & Burns, 2000). Minding these differences in acquisition, previous research has set the standards for reading and mathematics instruction, placing the development of concepts as a necessary precursor to more difficult skill acquisition. In other words, the development of these basic underlying concepts serves as the basis for later learning and achievement within reading and mathematics (Anthony & Lonigan, 2004; Campbell, Pungello, Miller-Johnson, Burchinal, & Ramey, 2001; Chaney, 1998; Gelman & Meck, 1983; Lonigan, Burgess, Anthony, & Barker, 1998; Resnick, 1989; Stevenson & Newman, 1986; Whitehurst & Lonigan, 2001). This idea of acquiring a sound foundation of skills prior to beginning higher-order learning is supported by federal and state law, research already completed in the area of early literacy, new research conducted by the National Mathematics Advisory Panel, and current research in early numeracy.

### *Federal Education Law*

*No Child Left Behind.* Many laws have been put into effect in order to regulate the education of American children. On January 8, 2002, President George W. Bush signed the No Child Left Behind Act. President Bush stated that the purpose of this Act is that, “All students have a better chance to learn, excel, and to live out their dreams.” This law is the re-authorization of the 1965 Elementary and Secondary Education Act, with the change being that the academic standards set by states are composed of clear, measurable

academic standards where schools are held accountable for the results of their students. In addition, the federal government supports both states and individual schools with greater resources. The overall goal of the No Child Left Behind Act is to close the achievement gap between disadvantaged and minority students from more advantaged students. Thus, according to this Act, all students will demonstrate measurable gains in reading and mathematics until they achieve at or above their grade level in these subjects (Spellings, 2007).

This law put into place several educational standards in order to reach the aforementioned goal. First, all schools must hire and employ only “highly qualified” teachers in all core subjects. A “highly qualified” teacher is defined as one who has at least a Bachelor’s Degree in teaching, state certification in that area, and the ability to demonstrate expertise in the subject area that they teach. Second, all schools need to use proven, research-based instructional methods when working with all students in their classes. Third, parents need to be provided with timely information regarding their child’s achievement as well as progress of their child’s school as a whole. Parents are required to be provided with options for their child’s education. Along with this provision, children’s needs are placed at the top of a school’s list of priorities. That is, the options of tutoring and/or transfer to other districts must be provided to parents and offered for all students who demonstrate low levels of achievement. Finally, parents will be provided with the Nation’s Report Card where they are able to compare the progress of their child’s school with the average of other schools around the country (Spellings, 2007).

So far, early data comparisons demonstrate that No Child Left Behind is working. According to National Assessment of Educational Progress assessments, nine-year-old students have demonstrated greater progress in reading between the years of 1999 and 2004 than nine-year-olds in the 28 years prior to this. In addition, the achievement gaps in reading and mathematics between African American and Hispanic nine-year-olds and their white counterparts have decreased dramatically. Also, national mathematics scores for 4<sup>th</sup> and 8<sup>th</sup> grade students are higher than ever before. It is important to note that some of the largest gains have been demonstrated in the most diverse urban schools in America. According to the National Assessment of Educational Progress Trial Urban District Assessments, fourth grade students in a majority of the sample urban schools made greater gains in reading and mathematics than students nationwide. For example, in New York State, fourth grade mathematics achievement increased by 11 points between the years of 2002 and 2004 while the achievement gap for African American and Hispanic students decreased by 10 points. In 2004, 70% of New York's fourth grade students met all state Learning Standards in English (Spellings, 2007).

*Response to Intervention.* As a result of greater demands for school district accountability in the No Child Left Behind Act, an education trend has developed to utilize research-based interventions early in a child's academic career in an attempt to remediate academic deficits. Educators call this trend Response to Intervention (RTI), and it has now become part of the federal and state legislation for classifying students with a possible learning disability.

The Individuals with Disabilities Education Improvement Act (2004) is the federal law that guarantees that children with disabilities are provided with a Free and

Appropriate Education designed to meet their individual needs. This law has recently been reauthorized and is considered The Individuals with Disabilities Education Improvement Act (IDEA) of 2004, (34 CFR § 300, 2006). The reauthorization of this act brought about changes regarding policy on the classification of individuals with disabilities. Section 300.307, regarding Specific Learning Disabilities, has been revised to state that, “The criteria adopted by the State must permit the use of a process based on the child’s response to scientific, research-based intervention” but cannot prohibit the use of the severe discrepancy model to classify a student with a Specific Learning Disability. This means that although students can still be identified as a student with a disability by demonstrating a significant discrepancy between their cognitive ability and academic achievement (measured by an IQ test and standardized academic achievement tests), states may also consider a student’s history of their response to research-based interventions as criteria for classification.

In response to the federal law IDEA, each state has developed their own guidelines regarding changes in general education classroom instructional procedures as well as learning disability classification. For example, the Regulations of the New York State Commissioner of Education Relating to General Education and Diploma Requirements have been amended to address this change in educational practice (8 NYCRR § 100.2, 2007). Section 100.2 addresses Response to Intervention by defining minimum requirements for school districts regarding this policy. First, school districts are now required to provide appropriate instruction to all students in general education, which includes research-based methodologies to meet the needs of a diverse group of students. Second, all students must undergo a screening process designed to identify

students who are not demonstrating adequate academic progress. Third, in order to meet the needs of the students who are not making adequate progress, instruction must include increasingly intensive levels of targeted interventions and instruction. Fourth, school districts should utilize repeated assessments of student achievement in order to measure the effectiveness of the targeted interventions. These repeated assessments should include curriculum based measurement to determine if the interventions are successful at increasing student achievement. Fifth, information regarding student progress should be documented and used to make educationally-based decisions regarding changes in academic goals, instruction, services, and referrals to the multidisciplinary team for the classification of students with learning disabilities. Finally, parents are required to be notified regarding the need for intervention beyond what is typically offered in the general education classroom, with documentation of progress and information regarding specific interventions. In addition to these types of requirements in schools across the country, school districts are now required to define the specific structure of their Response to Intervention programs regarding the levels of intervention provided to students, types of interventions offered, and the procedure and frequency of progress monitoring. This also includes educating staff in order to appropriately implement an effective Response to Intervention program.

The Response to Intervention model was spurred by educators who found the traditional discrepancy model of identifying students with Specific Learning Disabilities to be unsatisfactory. The discrepancy model states that an Individualized Education Program team may classify a student as learning disabled if a severe discrepancy exists between intellectual ability and academic achievement within the areas of oral

expression, listening comprehension, written expression, basic reading skills, reading comprehension, mathematics calculation, and/or mathematics reasoning (The Individuals with Disabilities Education Improvement Act of 2004, 34 CFR § 300.541). The longer that this process was in place, the more educators began to see problems with the discrepancy model. First, each state was able to decide how to operationalize the discrepancy between ability and achievement. However, due to inconsistencies across states, there are no clear criteria regarding how a student is identified as a student with a learning disability. Next, it is believed that these eligibility criteria leave many students with unidentified learning disabilities with no academic assistance until eventually, either a discrepancy becomes large enough to warrant services or the student continues to experience extreme difficulty and high levels of failure. Finally, the discrepancy model does not provide the Individualized Education Program team with relevant information to plan academic instruction to remediate a problem (Bradley, Danielson, & Doolittle, 2005).

The Response to Intervention model can best be described as the process of being provided with appropriate, research-based educational methodologies, monitoring student progress on a regular basis, providing supplemental instructional interventions for students who are not demonstrating appropriate educational progress, and finally, for students who do not respond to the interventions, considering special education services (Fuchs, Mock, Morgan, & Young, 2003). Given this model, supporters of Response to Intervention believe that the most effective way to make decisions regarding identification of specific learning disabilities is by utilizing the measurement of a student's unique responses to a series of research-based interventions based on problem-

solving regarding that student's unique strengths and weaknesses within the classroom context (Bradley et al., 2005).

The Response to Intervention model can be conceptualized as a three-tiered model. The first tier of this model includes the general education classroom. All students are considered to be in the first tier of RTI, as the interventions within this tier are provided for all students. This includes the universal screening procedures and progress monitoring of regular classroom instruction. The second tier of instruction includes small group interventions for students who do not make adequate academic progress when exposed only to the first tier, or the general education curriculum. These group interventions are specific to student needs and are research driven, and are targeted to last for a specific duration. Finally, the third tier is for a small number of students who still do not demonstrate appropriate progress even with the addition of tier two interventions. Students within the third tier receive additional interventions, however, these differ from the second tier of interventions in that they are individualized and more intensive (Bradley et al., 2005).

*Support of the Response to Intervention model.* There has been some recent research to support this model with young children. Authors Vellutino, Scanlon, Small, and Fanuele (2006) sought to find the effects of interventions on children who were identified with reading difficulties upon entry to kindergarten. These researchers conducted a five year longitudinal study to follow students who received reading intervention services in kindergarten and/or first grade. Children were identified as poor readers by their performance on a test of letter-name knowledge upon entrance into kindergarten. Once identified as a poor reader, these children also completed tests of

phonological awareness, rapid automatized naming of objects, counting by ones, and number identification.

From this screening procedure, the children who were identified as poor readers were randomly divided into two groups that can be thought of as the treatment group and the control group. The control group was used for comparing the students who underwent reading interventions to those who did not receive any form of intervention. Students in the treatment group were provided with small group instruction that lasted for 30 minutes two times per week throughout their kindergarten school year. This instruction was provided by a certified teacher, and children received this intervention in groups of two or three students. Instruction focused on emergent literacy skills including print concepts, print awareness, letter identification, phonological awareness, letter-sound mapping, sight word reading, and guided reading. Throughout the year, children were readministered the letter identification and phonological awareness tests that they first took during the screening process. In addition, all students were assessed on print concepts, word identification, knowledge of letter sounds, decoding, phoneme blending, and phoneme segmentation. Results demonstrated that overall, the group differences between treatment and control groups were statistically significant in many areas. The treatment group performed better than the control group on tests regarding phoneme segmentation, letter names, letter sounds, word identification, and decoding.

Once these children entered first grade, they were reevaluated in order to determine who continued to need remedial reading services. Children completed the Letter Identification, Word Identification, and Word Attack subtests from the Woodcock Reading Mastery Tests – Revised along with the letter sound, word identification, and

decoding screening measures completed in kindergarten. From these tests, students were divided into four groups: students identified as poor readers in kindergarten who were “no longer at risk,” poor readers at the beginning of kindergarten who continued to require remedial reading services, poor readers identified at the beginning of first grade, and normally achieving readers. The first grade intervention consisted of two experimental conditions. The first condition focused on the development of phonological skills. In the second condition, interventions were targeted at developing text processing skills such as code and meaning based strategies and comprehension monitoring. Each of these conditions had both of these kinds of activities, with the focus differing depending on the needs of the group. Also, these intervention groups also engaged in sight word identification and writing activities. Measures of reading achievement were then administered at the end of first, second, and third grades.

Results of this study suggest that these small group low intensity interventions are effective in remediating a large percentage of poor readers. At the end of third grade, 84% of the children who were identified as poor readers and received remedial services in either kindergarten or both kindergarten and first grade performed within the average range on reading achievement tests. Of this group, 73% of the children had only received intervention services in Kindergarten, which is 62% of the overall sample (poor and average readers combined). In addition, of the students who received intervention services in both kindergarten and first grades, 58% performed within the average range on achievement tests at the end of first, second, and third grades. Thus, this information suggests that early identification of poor performance in reading is an important first step in remediation services in order to catch up to their average performing peers. Although

this study was completed regarding reading performance, it is likely that the same would apply for mathematics.

In another article, a different approach was taken to determine the effects of RTI. Researchers VanDerHeyden, Witt, and Gilbertson (2007) were interested in the effects of the Response to Intervention model on the number of students identified for special education. In this study, the System to Enhance Educational Performance (STEEP) was evaluated with regard to its effects on the total number of special education evaluations, the percentage of those evaluations which qualified students for special education services, any reductions in assessment and placement costs for the district regarding special education services, and the outcomes of children who did not respond adequately to this RTI process.

During this study, curriculum-based assessment probes were administered to all students twice per year to measure reading and mathematics achievement for grades three to five. Then, after this data was collected, more probes were administered each month in order to track the class progress until the median fell within the mastery range. After the initial assessment, teachers received information pertaining to whether there are any problems that existed classwide or in individual students in either reading or math. If a classwide problem was identified, the classroom teacher began administering easier curriculum-based measurement probes until the instructional range was found for that particular class. The teacher then began to administer interventions for 10 minutes each day for 10 consecutive school days. Interventions most often consisted of modeling the target skill, guided practice with immediate feedback, timed independent practice for data collection, and use of a delayed error correction with verbal rehearsal. After the 10<sup>th</sup> day,

students who performed below the 16<sup>th</sup> percentile in their classes were moved on to the next more restrictive stage, where they received individual interventions. During the individual interventions, the same process as above was followed, with the addition of a motivational game where children attempted to “beat their high score.”

Results of this study demonstrate that this specific Response to Intervention process was effective in many different ways. First, there was a significant decrease in the number of initial evaluations during the STEEP implementation. Overall, the number of referrals at all of the schools combined during this RTI program was reduced by more than 50%. Second, there was an increase in the number of appropriate referrals for special education assessments. That is, during the baseline years, an average of 51.75% of students tested for special education services qualified for those services within this district. During the implementation of the STEEP program, this percentage of students referred to special education who qualified for these services increased to 69.5%. Finally, some decreases were found within the cost of districts regarding psycho-educational testing and special education services. It is hypothesized by these researchers that this number is not as high as expected because other services are offered in lieu of traditional testing. There was additional curriculum based measurement conducted, and interventions for many students which offset the costs of assessment. However, placement costs were reduced during STEEP implementation. Fewer students were being placed in special education classes, and for this particular district, four resource teacher positions were cut. However, in place of these positions, intervention specialists were hired to take on the increased number of intervention referrals at these schools. So, in this case, the money saved was reallocated to create other more appropriate teaching

positions within the district. So again, it does appear that RTI programs in schools are cost-effective and actually increase student achievement.

### *Early Literacy*

A well-developed foundation of research exists within the area of emergent literacy that leads to an appropriate application of the Response to Intervention trend. This research has defined the pre-reading skills that are important for later reading development. The early skills that are currently believed to be the most important to reading development are phonological sensitivity/awareness, oral language skills, and an understanding of the conventions of print (Anthony & Lonigan, 2004; Chaney, 1998; Lonigan et al., 1998; Whitehurst & Lonigan, 2001). In addition, these skills may be best conceptualized as a continuum upon which emergent reading skills begin and later literacy skills emerge, with no clear beginning or ending points (Lonigan et al., 2000). A brief explanation of these pre-reading skills follows.

Phonological sensitivity/awareness is an umbrella term for many different skills that begin to be acquired before children are exposed to formal reading instruction in the schools. During its early stages of development, phonological sensitivity can be observed when children detect large phonological units such as words, syllables, onsets, and rimes (Anthony & Lonigan, 2004). Phonological sensitivity abilities are further revealed through tasks that require the child to rhyme, match initial consonants, or count the number of phonemes that are apparent in spoken words (Stahl & Murray, 1994). In later stages, these skills are further developed such that children are able to demonstrate the metalinguistic ability to manipulate phonemes (Anthony & Lonigan, 2004). For

example, by middle childhood, many students possess the ability to break words down into phonemes and explain how one word is different from another.

These early childhood abilities, understood as phonological sensitivity, have been shown to have the greatest impact on later reading achievement in children. For example, MacLean, Bryant, and Bradley (1987) found that three-year-old children's knowledge of nursery rhymes was correlated with the later development of abstract phonological skills. Also, children who have well-developed skills in understanding syllables, rhymes, and phonemes learn to read more quickly than students who have not yet developed these skills before formal reading instruction (Stahl & Murray, 1994). These skills have been shown to be important even after controlling for intellectual level, vocabulary, memory span, and socioeconomic status (Anthony & Lonigan, 2004). In summary, the phonological skills that are developed before formal reading instruction set the foundation for more advanced reading skill development. These skills furthermore reflect the same underlying abilities that are evident when children are able to think about phonemes from a metalinguistic standpoint (Anthony & Lonigan, 2004).

In addition to phonological sensitivity, oral language skills have recently been found to play an important role in reading development. Oral language skills typically include vocabulary development, understanding syntax, being able to participate in discourse, and the earliest forms of phonological awareness. Of these abilities, vocabulary knowledge has been found to be the most influential factor in later reading ability after taking phonological awareness into account (Dickinson, McCabe, Anastasopoulos, Peisner-Feinberg, & Poe, 2003). When children are first developing their vocabulary knowledge, speaking to and listening to others are important activities

that later grow into skills. These types of vocabulary skills have been shown to have a significant impact on the decoding of letters into sounds and then letters into words (Wagner & Torgesen, 1997). Therefore, similar to phonological sensitivity, the early development of oral language skills impacts later literacy progress.

Finally, a child's understanding of the conventions of print has been shown to be an important precursor to later reading ability. Conventions of print include the basic rules of reading. For instance, one needs to know that you must read left to right and top to bottom in order for the print to make sense. Also included in conventions of print are reading from the front to the back of a book, determining between the covers and pages of a book, determining between pictures and words on a page, and understanding punctuation (Lonigan et al., 2000; Whitehurst & Lonigan, 1998). Results of a study completed by Tunmer, Herriman, and Nesdale (1988) suggest that understanding the abovementioned concepts about print in preschool and kindergarten can predict reading comprehension and decoding abilities that will develop by the end of the second grade.

All of these abilities, phonological sensitivity, oral reading skills, and understanding conventions of print, have a strong foundation in the research literature concerning emerging literacy development. While these abilities and concepts are somewhat distinguishable from one another, it is important to know that each of these are also interrelated (Lonigan et al., 2000; Whitehurst & Lonigan, 1998). Any poor reading skills can negatively impact later learning in reading. It is also possible that these skills may have an influence on learning in other subject areas that have at least some basis in reading (Lonigan et al., 2000).

From the information obtained by researchers in the area of reading, some curriculum-based measures have been developed to screen skill acquisition in the various areas of early literacy development. These measures meet the criteria of the Response to Intervention model and are appropriate for early intervention with students. One such measure is the Dynamic Indicators of Basic Early Literacy Skills (DIBELS). The DIBELS were developed by researchers Good and Kaminski (1996) and were based on literature regarding curriculum-based measurement by researchers Deno (1985), Deno and Fuchs (1987), Deno and Mirkin (1977), and Shinn (1989). The result is a set of quick, one minute fluency tasks to measure and monitor early literacy and reading skills in children grades Kindergarten through 6<sup>th</sup> grade. Schools are able to interpret performance on these tasks in order to make decisions regarding individual or group progress and educational instruction of students (University of Oregon Center on Teaching and Learning, n.d.).

Coyne and Harn (2006) describe how the assessment of early literacy skills can lead to improved instruction and intervention, thus encouraging better acquisition of early reading skills in all students. According to these researchers, DIBELS are a comprehensive assessment system that is designed to assess the specific components of early literacy: phonemic awareness, alphabetic understanding, fluency, vocabulary, and reading comprehension.

These researchers promote the use of DIBELS for the purposes of screening, progress monitoring, diagnosis, and measurement of student outcomes. First, as a screening tool, all students are given the assessment early in the year. Then, school professionals can determine which students are considered at-risk. If a student is

identified as at-risk, it means that their skills are less developed than those of their peers and that they are most likely to experience reading difficulties in the future.

Once the children are screened and determined to be at-risk, school professional can design individualized instruction programs to increase student achievement. These students then move to the next step, progress monitoring. In the progress monitoring phase, schools continue to assess the at-risk students in order to determine if they are making progress toward their reading goals. The results obtained from this assessment provide information regarding the students' learning. If the student is determined to be making adequate progress, they can continue on their current plan. However, if a student is determined to be making adequate progress, they can continue on their current plan. However, if a student is determined not to be making adequate progress, new instructional techniques can be put into place.

In addition to screening and progress monitoring, DIBELS assessments allow for diagnostic decisions to be made. For example, the specific skills that students are lacking are identified through this process. Then, the students are monitored with regard to their progress on specifically designed interventions. The way that students respond to interventions provides important information regarding how they learn and what type of instruction is most effective for that individual student.

Finally, Coyne and Harn (2006) note that DIBELS assessments can be used for measuring student outcomes. That is, by measuring student progress in literacy skill acquisition, educators gain information on individual student performance as well as the overall reading program. Outcome assessments provide information regarding individual

student performance, grade-level performance, overall school performance, and the relationships of those performance levels among one another or to a national sample.

To summarize, these researchers identify the DIBELS as a literacy measurement tool that can be used for many different reasons in schools. Those reasons might include screening, progress monitoring, diagnosis, and measurement of student outcomes.

Regardless of the reason, DIBELS tasks provide appropriate measurement of literacy skill acquisition for early intervention with students who are at-risk for developing reading difficulties.

#### *Early Skills Related to Later Academic Achievement*

While research on the effects of pre-reading skills on later academic achievement in reading is thorough and well developed, there is also evidence to suggest that early skills predict academic achievement throughout a child's school career, as well as several years past kindergarten and first grade. Researchers Stevenson and Newman (1986) found a correlation of 0.46 between preschool mathematics achievement and tenth grade mathematics performance. In this study, children were administered the arithmetic subtest of the Wide Range Achievement Test (WRAT) in the summer prior to entering Kindergarten. At this age level, the arithmetic test consists of counting and recognizing and reading number symbols. When tested again at the fifth grade and tenth grade levels, children are expected to also complete oral and written computation tasks. These researchers found the performance on the WRAT in preschool was significantly correlated to performance in fifth and tenth grades on the same test at the .01 significance level. At fifth grade, these researchers identified a 0.5 correlation and then a .46 correlation in tenth grade. Also, these children were administered a problem-solving task

that was developed by identifying problem solving problems in tenth grade mathematics curricula. Preschool math scores on the WRAT were found to have a significant .52 correlation with those tenth grade problem-solving tasks.

Possibly more relevant to mathematics assessment today, researchers Locuniak and Jordan (2008) completed a study examining how number sense in kindergarten students was related to second grade calculation fluency. In kindergarten, these children were administered tests of number sense including counting, number knowledge, nonverbal calculation, story problems, and number combinations. The counting task required the children to demonstrate enumeration, count in sequence, number recognition, and demonstrate their understanding of main counting principles. For the number knowledge task, the children were shown a number and asked which number came before and after. Nonverbal calculation included children watching addition and subtraction problems take place on a mat in front of them with chips. For the story problems and number combinations tasks, children were administered oral questions related to addition and subtraction tasks. In addition, these students were administered letter-naming fluency, phoneme segmentation fluency, and nonsense-word fluency tests from the Dynamic Indicators of Basic Early Literacy Skills (DIBELS), and a memory span test from the Wechsler Intelligence Scale for Children – Fourth Edition (WISC-IV). When in second grade, the children were administered the Assessment of Math Fact Fluency test developed by Fuchs, Hamlett, & Powell in 2003. This test includes a fluency test of basic addition and subtraction calculation tasks timed for one minute.

Locuniak and Jordan found that early number sense skills did indeed predict later calculation fluency skills in second grade. Even more, these researchers found that

kindergarten number sense predicted calculation fluency even more than did predictors of age, reading, vocabulary, and memory. With regard to specific tasks, those which had the highest correlations with calculation fluency in second grade were counting (.30), working memory (.35), number knowledge (.45), nonverbal calculations (.51), story problems (.51), and number combinations (.57). While these researchers based their research on specific hypotheses for early skills, the National Mathematics Advisory Panel completed a thorough examination of the literature and best practices in mathematics to identify the specific precursors to later mathematics achievement, specifically in algebra.

*National Mathematics Advisory Panel*

Recently, the U.S. Department of Education formed the National Mathematics Advisory Panel in order to complete research that demonstrates the importance of mathematics education as well as to promote “Greater knowledge and improved performance in mathematics among American students” (National Mathematics Advisory Panel, 2008). This panel finds that currently, mathematics achievement among United States students is lower than that of its peers in other areas of the world. Although our “National Report Card” demonstrates positive growth on mathematics assessments at grades 4 and 8, only 32% of 8<sup>th</sup> grade students are considered at or above the “proficient” level and only 23% of our students are at or above this level in grade 12. While our minority race and income populations continue to grow, the achievement gap in the area of mathematics continues to be an evident area of inequality.

The overall conclusion that this Panel has reached is that the current mathematics education of our students is inadequate for the purpose of training and preparing active members of society. They found that there was a need for mathematics curriculum in

grades pre-kindergarten through eight to focus on the most critical foundations for learning algebra. The Panel summarized the Critical Foundations of Algebra into three categories of skills: Fluency with Whole Numbers, Fluency with Fractions, and Particular Aspects of Geometry and Measurement. These Critical Foundations are considered to be the skills necessary to be able to adequately learn algebraic concepts. While these categories provide a structure by which to organize curriculum, the Panel emphasizes that this should be thought of as a guideline to which other concepts should be added.

The first category, Fluency with Whole Numbers, poses that by the end of Grade 5 or 6, children should have mastered their sense of number. This number sense includes understanding place value, the basic operations (addition, subtraction, multiplication, and division), commutative, associative, and distributive properties, computation, and the applications of operations to problem solving.

The second category is Fluency with Fractions. The Panel recommends that Middle School students should have acquired a solid foundation in the concepts of fractions. According to the Panel, students should be able to “locate positive and negative fractions on a number line, represent and compare fractions, decimals, and related percentages, and estimate size.” They should also be able to compute addition, subtraction, multiplication, and division of fractions.

The third category is considered Particular Aspects of Geometry and Measurement. Within this category are the skills of computing the perimeter and area of triangles and quadrilaterals, the ability to analyze the properties of two- and three-dimensional shapes, and solving problems related to surface area and volume. By the end of Grade 7, students should begin to understand the concept of slope.

The Panel also makes several recommendations for “Readiness for Learning.” The Panel begins by stating that children begin learning far before entering school. That information is built upon within the formal school setting, beginning in Kindergarten. With regard to this timeframe, the Panel suggests that research is necessary in order to gain an understanding of children’s mathematics capabilities, and that the area of mathematics education and skill acquisition is lacking peer-reviewed research. However, what information is available in mathematics suggests that conceptual knowledge, procedural skills, and the commitment of math facts to long-term memory are the basis of mathematics learning. In addition, these skills each affect the development of number sense, understanding of fractions, and geometry and measurement concepts, which are the basis of arithmetical and algebraic concepts.

Number sense begins to develop when children are in their preschool years and continues on through their educational careers in such concepts as fractions, decimals, percentages, and exponents. These skills can be considered an understanding of numerical values associated with quantities, counting, and approximation of magnitude. Many American children demonstrate difficulty acquiring appropriate skills using fractions. This may be due to students having a poor skill base in retrieving basic math facts and understanding whole numbers. Finally, the Panel has found that many children have a poor understanding of geometry and measurement. While they found that children are often taught these concepts by using concrete and tangible concepts, they are unable to develop abstract representations necessary for some algebraic concepts. Recommendations of the Panel in these areas include that children learn appropriate methods of estimation and are taught conceptual knowledge of fractions through the use

of representational supports (e.g., number lines) in order to gain a greater understanding of the procedural techniques when solving math problems with fractions.

In summary, the Panel found that higher order learning in mathematics is based upon the foundation of skills needed in order to understand algebra. While many gaps in the literature exist, it is evident that mathematical learning begins early, even prior to formal instruction in kindergarten. Young children are learning about mathematical concepts prior to beginning school by observing and modeling others as well as through play. This idea is demonstrated in mathematical literature which supports that play has direct cognitive implications for students (Ginsberg, Inoue, & Seo, 1999; Ginsburg & Seo, 2000; Henniger, 1987; Stannard, Wolfgang, Jones, & Phelps, 2001; Wolfgang, Stannard, & Jones, 2001).

#### *Early Mathematics: Mathematical Play*

One researcher, Henniger (1987), demonstrated several examples of mathematics play in preschool children. Henniger operates under the assumption that the earlier children are exposed to mathematics, the more likely they are to develop a conceptual understanding of mathematics. Mathematical experiences that are typical of school-age children are not appropriate for preschool students. Instead, those experiences are best obtained through play.

Henniger's (1987) study focused upon many different examples of mathematical play. While the children were playing in the sandbox, they were often observed filling and emptying containers. When children engaged in this activity, they demonstrated the concepts of measuring volume by filling up different types of containers. These children were also counting the number of spoonfuls of sand that were required to fill the

containers, which demonstrates the concept of ordinality. When the children in this study began to build a sand castle, the walls kept collapsing. When this occurred, the children experimented with different ideas until they figured out the appropriate texture to keep the walls standing up, demonstrating the problem solving skills that they will utilize in later mathematical concepts. In the garden section, children were observed to sort seeds by size and shape, demonstrating the concept of classification.

In the block play area, children were observed building different forts. When they did so, they used blocks of different lengths and shapes, again demonstrating classification skills. Also, these children were able to learn that when no more of the large (24 inch) blocks were left to build with, two smaller (12 inch) blocks covered the same amount of area. This can be considered an informal addition problem. Children were observed comparing cylindrical and rectangular shapes, which was a comparison of surface area and volume. Finally, within this specific area, children often counted blocks while they built or when they were finding materials to use while building.

Overall, this researcher noticed that children who were utilizing mathematical play developed some interesting attitudes towards math. First, children were curious and interested in learning these concepts. Second, these children were demonstrating problem-solving techniques by using divergent thinking in their play. Finally, the motivation of these children to learn is increased during play in comparison to specific lecture style teaching methods due to the child choosing what to play and engaging in hands on learning.

Researchers Ginsburg et al. (1999) also set out to identify the mathematical concepts and skills that preschoolers utilize through play. In order to obtain this data,

these researchers videotaped children's play for 15 minute intervals during free play activities. Subjects included 11 four-year-old and 19 five-year-old students. These intervals were then analyzed using two different methods: deep analysis to provide information regarding the individual child's cognitive abilities (specific), and surface analysis for comparing common mathematical concepts across children. During the surface analysis, researchers coded for behaviors related to classification, relations, enumeration, dynamics, patterns, and shapes.

During analysis, the researchers found that children engaged in many different mathematical activities. One group of children was observed counting beads. Although only one student was truly counting the beads by picking them up and putting them in his hand, other children joined in until they reached 100 beads. Another child demonstrated complex geometric knowledge during block play.

These examiners determined the amount of time that the students spent engaged in mathematical activities. Of the time observed, 44.6% of children's free play activity was engaged in mathematical activity. This activity occurred most frequently during construction and pattern play. That is, 44.6% of their play was spent engaged in activities regarding classification, relations, enumeration, dynamics, patterns, or shapes. Of this time, 36% of the mathematical engagement was work with patterns and shapes (shape/pattern detection, prediction, or creation), 22% of the time was spent on dynamics (exploration of the processes of change or transformation), 18% on relations (magnitude evaluation or comparison), 13% on classification (sorting, grouping, categorizing), and 11% on enumeration (quantification or numerical judgment). Children engaged in

mathematical activities while playing with puzzles, clay/sand/water tables, legos and blocks, and during competitive, cooperative, silent parallel play, and verbal parallel play.

Researchers Wolfgang et al. (2001) added to the conceptualization of block play as mathematics play. These researchers identified many areas of learning that take place during this type of play. They defined block play as using a high number of objects in order to spatially represent other larger objects that children encounter in their lives. That is, children represent real objects with their imaginary structures. When playing with blocks, children use classification, measurement, ordering, counting, and the use of fractions. In addition, children become more aware of depth, width, and length, symmetry, shape, and the occupation of space. Each of these concepts is believed to be linked to later conceptual learning in mathematics lessons.

Researchers Stannard et al. (2001) analyzed the school performance of 37 students who attended a play-based childcare program from the age of 2 through Kindergarten. These children's levels of play were assessed using the Linzer Five Point Play Scale on levels of adaptiveness, complexity, and insight in each area of play. The areas assessed included block play and lego play where they were told to build what they want with as many blocks as they needed, for as long as they'd like, and carpentry where students were given a variety of pieces of wood, nails, safety goggles, hammers, bottle caps, and markers for drawing and given the same directions as in block and lego play. The students were also assessed by the McCarthy Scales of Mental Abilities in order to obtain an IQ score. Students' mathematics achievement was later assessed by the California Achievement test in grades 3, 5, and 7 as well as high school mathematics class grades in grades 9, 10, 11, and 12.

Results of this study demonstrate that children's block play predicted mathematics achievement in 7<sup>th</sup> grade as well as in high school. In addition, Legos play and carpentry also predicted math achievement in 7<sup>th</sup> grade and high school as well as the number of high school mathematics classes taken. This information suggests that construction play and formal mathematics skills require procedure commonalities in relation to a student's developmental stage. That is, experiences in the preschool years cumulatively affect adolescent cognitive behavior in Piaget's formal operational stage of development.

Another group of researchers studied how young children utilize mathematical concepts in everyday language and play (Ginsburg & Seo, 2000). These researchers observed four- and five-year old students during play and analyzed their language for evidence of mathematical concepts. The children engaged in several different activities during their free-play period that demonstrated mathematical understanding or learning. While playing "school" the children read a book and discussed the distance concepts of near and far as well as quantity and magnitude. This type of typical interaction among preschool age children (pretending to read a book and discussing the pictures) illustrates how they utilize mathematical concepts in language. This article provides evidence that children are learning mathematical concepts, even with little or no formal training.

#### *Early Mathematics: Concepts and Skills*

Within the literature in early childhood mathematics, there tend to be two areas of interest: Counting concepts and number sense. Counting concepts can be thought of as understanding five basic principles in order to learn to count. Number sense is characterized by the acquisition of skills that are needed in order to build mathematical knowledge. Within the literature, these concepts and skills have been correlated with

later mathematics performance scores up to tenth grade (Locuniak & Jordan, 2008; Stevenson & Newman, 1986).

*Counting concepts.* Researchers Gelman and Meck (1983) felt that children ages 3 to 5 years old possess a great deal of mathematical knowledge that is not demonstrated in typical performance-based assessments. That is, many young children who are unable to count large sets of numbers or perform addition and subtraction problems still possess the implicit knowledge of the counting principles one-to-one correspondence, stable-order principle, cardinality, abstraction, and order-irrelevance. These researchers sought to demonstrate this implicit knowledge through a series of error detection studies. During these studies, children observed a puppet count, and then told the examiner whether the puppet counted correctly or incorrectly. Gelman and Meck believed that this would result in a truer estimate of mathematical ability, given that they only had to observe the skill, not generate and monitor their own counting performance. These studies were conducted with regard to one-to-one correspondence, the stable order principle, and cardinality.

During the one-to-one correspondence study, 12 three-year-olds and 12 four-year-olds were exposed to six counting trials consisting of either correct, in-error, or pseudo-error sequences. The in-error trials consisted of either skipping an item during counting or double counting an item. The pseudo-error sequences included beginning counting of a linear array in the middle, correctly counting to the end of the array, and then returning to the beginning of the array and correctly finishing the counting sequence to the first item counted or counting a row of alternating red and blue chips by first counting all of the red chips and then returning to the first blue chip and correctly counting the rest of the

blue chips. The pseudo-error trials were actually correct ways of counting, but counting in a non-traditional way. For this particular study, the children were administered the pseudo-error items for use in another study. Because these items were not analyzed as part of this study, they will not be discussed further. With regard to the in-error and correct sequences, 75% of the three-year-olds and 83% of the four-year-olds were able to identify when the puppet was making an error, and a majority of the children in each group were able to identify the error and explain why it was an incorrect way of counting. However, it is important to note that age effects existed where four-year-olds were able to identify more errors than the three-year-olds.

Gelman and Meck (1983) also examined the stable-order principle with each child being exposed to five trials per set size, where each set included 5, 7, 12, or 20 items in a row. During each of the five trials, children were exposed to two correct and three incorrect sequences with regard to the stable-order principle. The three types of errors encountered by the children included a reversal of two items, use of a randomly ordered list of numbers, and a list where one or more tags are skipped in standard counting. Subjects for this study included 12 three-year-olds, 12 four-year-olds, and 12 five-year-olds. A majority of students were able to identify when the puppet counted correctly, and no age effects occurred for this measure. With regard to the error trials, age effects were noted. Three-year-olds demonstrated more difficulty than the four- and five-year-olds on identifying the stable-order errors. They were least effective in identifying trials where number names were omitted, identifying only 60% of these trials correctly. However, three-year-olds identified 80% of reversals and 90% of random strings of numbers. There was no significant difference between four- and five-year-olds on error trials.

Finally, the concept of cardinality was assessed with 12-three year-olds and 12 four-year-olds (Gelman & Meck, 1983). These children observed a puppet counting an array of items. At the end, the puppet asked the child, “How many?” The children were then asked to determine whether or not the puppet’s answer was correct. If the puppet counted incorrectly, the children were asked to correct the puppet. Out of five trials, the puppet was correct two times and incorrect for three of the trials. Three year-old children were able to identify errors 86% of the time and correct answers 96% of the time. Four-year-olds were able to identify errors 99% of the time and correct trials 100% of the time. When asked to correct the puppet, three-year-olds did so 70% of the trials with 94% accuracy and four-year-olds answered for 90% of the trials with 93% accuracy. Significant age effects existed between three- and four-year-olds on this task, however, both age groups were able to identify errors and explain why the puppet as in error, demonstrating that they do possess the implicit knowledge of cardinality.

Researchers Briars and Siegler (1984) examined preschoolers’ counting abilities with regard to the Order Irrelevance and Abstraction principles. Participants included 10 three-year-olds, 10 four-year-olds, and 10 five-year-olds from a middle class preschool. Students were presented with cardboard strips with a row of ten plastic chips of alternating colors pasted onto them and given examples of counting by a puppet. The children were presented with three types of problems. These types were standard counts, counting errors, and unusual counts. The counting errors included violating word/object correspondence by assigning too many or too few number words to one of the objects, omitting a word by pointing to an object but not saying anything, or skipping objects altogether. The unusual counts included counting from right to left, starting in the middle

of the row and finishing on the other side of that object, and counting every other object until they reach the end of the array, and then reversing the counting back the opposite way that they started to complete counting the objects that have not yet been counted. The one conventional type of counting included correctly pointing to and counting every object from left to right. The purpose of this study was to examine students' understanding of the Order Irrelevance and Abstraction principles first presented by Gelman and Gallistel (1978).

The children were presented with the array of chips on poster board and introduced to a puppet named Scruffy. They were told that Scruffy was just learning to count and needed help in finding his mistakes. They watched Scruffy count the chips in the methods described above, and were then asked if he made any mistakes.

The results of this study demonstrated that with age, preschool children were able to understand that counting only requires assigning one number to each object, and can be counted in any order. While the three-year-old and some of the younger four-year-old children incorrectly reported that starting at a particular end or counting adjacent objects is a necessary component to correct counting sequences, the older four-year-olds and five-year-olds were more likely to accept the unusual forms of counting as correct.

*Number sense.* Besides having the ability to understand the concepts behind the skill of counting, researchers have identified other preschool-age skills with regard to mathematical abilities. These skills include number identification, counting skill, comparisons of magnitude, and completing informal addition and subtraction tasks. While the first two skills, number identification and counting, are rather well-developed, the last two are less developed in mathematical literature.

Researchers Bertelli, Joanni, and Martlew (1998) completed research regarding preschool age children's ability to count and reason about number. In order to do so, they recruited sixty children ages 3 to 4. These children participated in one of two different nursery programs. In the first (Group A), the aim was to prepare children for entry to school by encouraging engagement in specific number activities in order to enter school at age five while those in the second group (Group B) were encouraged to engage in a number of tasks such as cutting, painting, gluing, and story time, but no specific number tasks. The children in the second group did not enter school until age seven. Although these two groups are from different cultures, their socio-economic status was similarly matched. These researchers developed their study in order to examine the development of mathematics skills and concepts in relation to their counting abilities.

The children in this study were presented with cut-out teddy bears wearing a shirt with buttonholes but no buttons on their shirts, and plates with buttons on them. The children were presented with eight types of changes regarding addition, subtraction, and the concept of conservation. The children were presented with a teddy bear and a plate of buttons and asked, "Has teddy got another button?" These questions were asked after the following four actions: when one button was added to the set (addition), the buttons were spread out by shaking the plate (spreading), one button was added to the set as the buttons were spread out on the plate (addition combined with spreading), and one button was added to the set while the buttons were put closer to one another (addition combined with condensing). The question, "Has teddy lost a button?" was asked after the following four actions: one button was taken from the set (subtraction), the buttons were put closer to one another on the plate (condensing), one button was taken from the set and the other

buttons were placed closer to one another (subtraction combined with condensing), and one button was removed while the other buttons were spread apart (subtraction combined with spreading). Each of these eight actions was completed for set sizes of 3 and 21 buttons in random order. After this was completed, the children were asked to count a set size containing five items.

Results for these studies demonstrate interesting effects. With regard to the addition task, children in both groups were able to give consistently correct responses for both set sizes when one button was added to each group. However, the three year-old students in Group A demonstrated more difficulty with this task. They were able to complete it 93% of the time with a set size of 3, but only 60% of the time with a set size of 21. These same results were demonstrated for the addition combined with spreading task. Both ages in Group B and four-year-olds in Group A demonstrated consistent results while the three-year-olds in Group A demonstrated 93% accuracy at set size 3 and 60% accuracy at set size 21. With regard to the addition combined with condensing task, both ages in Group B were able to complete this task with accuracy. Group A demonstrated a greater deal of difficulty with this task. Both ages were able to demonstrate consistent results at set size three, but demonstrated poorer performance on the set size twenty-one task. Only 47% of three-year-olds and 73% of four-year-olds were able to complete this question accurately. When the buttons were only spread apart on the plate, only four-year-olds in Group B were able to demonstrate consistent results, with 73% accuracy for three-year-olds. Those in Group A were more successful with the set size of three (60% accuracy for both groups) than in the set size of twenty-one (20% accuracy for three-year-olds and 53% accuracy for four-year-olds).

On each of the subtraction tasks, similar results were obtained. For the subtraction task where one button was removed and the subtraction with condensing task, both age groups in Group B and four-year-olds in Group A demonstrated consistent results. Three-year-old children in Group A were able to demonstrate accurate results at set size three (87% and 93% accuracy), but had more difficulty with set size twenty-one (60% accuracy for both tasks). On the subtraction combined with spreading task, the Group B children and the four-year-old Group A children were able to answer both set sizes accurately. Three-year-old children in Group A were able to accurately complete set size three questions, and answered set size twenty-one questions with only 47% accuracy. On the condensing only task, all of the children were able to complete these questions accurately with the exception of three-year-old Group A children for set size 21 (53% accuracy). Finally, when children were asked to count a series of five items, both age groups in Group A were able to consistently count to five whereas children in Group B demonstrated poorer performance. Only 13% of three-year-olds and 20% of four-year-olds were able to accurately complete this task.

These results indicate some important information. It appears, from these results, that while important, counting is not necessarily related to understanding concepts of number. In this study, children without formal training in mathematical tasks such as counting, categorizing, and judging sets, were able to complete informal addition and subtraction tasks with more accuracy than children who were formally trained in these methods. While children in Group A were able to count accurately, they demonstrated poorer performance on the tasks where they were given a mathematical task with the opposite perceptual representation (addition with condensing or subtraction with

spreading). Thus, the importance of counting on later mathematical performance may not be entirely accurate, and it is possible that other development in mathematics may play an important role.

Researchers Murray and Mayer (1988) completed a study in order to determine if the concept of magnitude is one of the readiness skills for mathematical development in preschool children. These researchers used comparison and sorting tasks in order to measure a child's understanding of magnitude.

In the first experiment, these researchers utilized a comparison task in order to measure a student's understanding of the concept of magnitude. First, children completed a counting task where the experimenter dropped grapes into a bag one at a time. The students were asked to count the grapes as they dropped into the bag. Next, students completed a comparison task where they were shown two identical stuffed animals with a bowl in front of them. Each child was then told, "Pretend that we give Teddy four grapes and that we give Freddy five grapes. Which bear has more grapes, the bear with four grapes or the bear with five grapes?" No concrete examples were used, but the children did have an opportunity to practice these trials with feedback regarding their accuracy.

In the second experiment, children participated in a sorting task in order to demonstrate their conceptual knowledge regarding magnitude. In this experiment, each child was first asked to count to 10. Next, the experimenter laid three different sized jars on a piece of poster board. The jars were small, medium, and large sizes, respectively. The children were asked to tell the examiner how the jars were different from one another and were also asked to name each of the jars. Finally, for the sorting task, children were

shown 9 number cards with the numbers 1 through 9 printed on them. The experimenter showed the child the card, read the number and then asked the child to repeat the number. The examiner then asked the child to put the card in front of the jar where it “went best.” This task was repeated one week following the first administration.

Results of these two studies indicate that four-year-old children were able to complete these tasks with more consistency and accuracy than three-year-olds. Overall, three-year-old students had a minor understanding of magnitude. They were generally able to identify that the number one is small and that the numbers eight and nine are large. However, these children had more difficulty with discriminating the numbers in between and where they fit along the continuum and understanding the concepts of more and less. Conversely, four-year-olds were much better at completing these tasks, understanding the difference between small and large, small and medium, and medium and large. All of these children were able to count to 10, and were also able to determine that the differences in the jars were with regard to size. They were also able to appropriately name the jars with small, medium, and large labels.

These researchers maintain that although preschool children are typically able to count to 10, which is expected of their age group in school curricula, this alone may not be an appropriate expectation of mathematics readiness skills. Instead, they expect that magnitude plays a role in readiness, along with counting skills.

Another researcher, Sophian (2000), also set about to determine at what age children are able to make appropriate judgments about magnitude. In this study, three-, four-, and five-year-olds were shown a stuffed “Cookie Monster” and told that he was always hungry and wanted to fill his belly with cookies. They were told that Cookie

Monster did not care how many cookies he got, as long as he got as much to eat as possible. They were then presented with pictures of cookies in different amounts and sizes. Some of the pictures showed number and amount agreement while others showed number and amount conflict. That is, some of the pictures that had the largest number of cookies also had the largest size on the page, while others consisted of a large number of cookies in a smaller amount on the page than the smaller number of cookies.

Results of this study demonstrate that children gain a greater understanding of magnitude as they age through the preschool years. Four-year-olds performed better than three-year olds on this task, and five-year-old students performed better than both four-year-olds and three-year-olds. By five years of age, most children were able to determine which picture had the greatest amount, regardless of the number of cookies on the page. However, the younger students were more likely to choose the picture that had the largest cookie on it, regardless of how many cookies the other picture illustrated. Overall, these children tended to choose by size rather than by counting the number of cookies. This result suggests that again, children are able to demonstrate underlying mathematical principles without relying on counting techniques.

Finally, with regard to informal addition and subtractions tasks, the author Resnick (1989) provides information into preschoolers' proto-quantitative reasoning schemas (ability to make perceptual comparisons of objects) and early counting knowledge. According to this author, mathematics skills can best be viewed from a Constructivist approach. That is, math is not directly or innately learned, but rather is "constructed" through specific instruction as well as every day experiences. Even more, this author believes that young children are able to understand mathematical principles,

but are often unable to verbalize or outwardly demonstrate those principles. For example, infants begin to demonstrate conceptual mathematical knowledge as early as around six months of age. Some infants can recognize differences in numbers of objects, presented visually, as well as compare size, although obviously, without the use of concrete labels.

In preschool, children begin to develop proto-quantitative schemas. These schemas are perceptual and not based on direct measurements. Preschoolers express quantity through the use of absolute labels such as “big,” “small,” “lots,” and “little.” They are able to make verbal comparisons of two items such as telling if one house is larger than another or if one cup has more liquid in it than another. Preschool children also begin to understand an increase/decrease schema. They can logically understand that if there is a set amount of something, and you add more of the same thing to that amount, you get more of the amount. Also, they can understand that if you take an amount away from a set amount, you end up with less of the object. Interestingly there are some situations where children are unable to complete these quantitative judgments. Specifically, if the addition or subtraction of an amount of something occurs out of the child’s sight, they are unable to demonstrate knowledge of the change in quantity. The third proto-quantitative schema relates to part-whole relationships. Preschool children can make judgments about the relationships between parts and wholes. That is, they know that two parts put together make a larger quantity than each of the parts separately. They also understand that if they take one large piece of something and cut it into equal pieces, they can put it back together to equal the original quantity. In addition, they know that a whole object is bigger than any of its pieces. In the past, researchers have believed

that children are unable to complete conservation and part-whole schema tasks as in Piagetian theory. However, if language is adapted in order to reflect a whole collection rather than individual pieces, children are able to complete these tasks. For example, saying a “forest” instead of “pine trees and oak trees.” Another example is “a whole pizza” rather than saying “all of the cheese pieces and all of the pieces with pepperoni.” This does not mean that preschool children are not limited in these skills. Preschoolers still lack basic measurement skills necessary to complete traditional Piagetian conservation and part-whole tasks. To a preschooler, if something looks longer or larger, despite where the line begins or the shape of the container that objects occupy. However, researchers believe that the proto-quantitative skills that preschoolers do possess, in addition to counting skills, set the stage for the understanding of many major number principles.

Counting can be considered the preface of making exact proto-quantitative judgments. The concepts that precede counting include one-to-one correspondence, stable order principle, cardinality, abstraction, and the order-irrelevance principle. After attaining this knowledge along with counting skills, children begin to combine number naming with proto-quantitative comparison schemas. Therefore, children are able to understand that number represent more, and others represent less. The instructional implications of this information are that the more exposure and practice with counting and proto-quantitative concepts that preschoolers receive, the more developed the basis for more complicated mathematics principles becomes.

In addition, authors Klein and Bisanz (2000) studied mathematical development in four-year-old preschool children. Their aim was to determine what types of

mathematical concepts four-year-olds understood, and what informational processes are necessary in order to complete mathematics processing. Specifically, these researchers examined a child's ability to complete informal addition and subtraction problems.

These researchers completed a study with 48 four-year-olds from five different preschools. These children first completed a counting a matching pretest. If they were unable to complete these tasks, testing was discontinued. The counting pretest consisted of counting a set of 10 chips. For the matching pretest, children were asked to match the number of chips that went into a box while the children observed them being added. After demonstrating the ability to do each of these tasks, the children moved on to completion of two-term addition and subtraction tasks.

Children completed twelve two-term, single digit problems. Half of these problems were addition and half were subtraction. The children watched the examiner set up an initial array of chips in front of the examiner. Next, the examiner covered that array with a box and then added or subtracted chips from the hidden array so that the children were able to observe this action. The children were then asked to match the same amount of chips on their mat as was under the box. After they completed their array, the box was removed so that the children could see if they matched although no feedback was provided regarding accuracy.

Results demonstrated that children are able to solve two-term addition and subtraction problems when the numbers are less than six. The researchers suggest that this is due to constraints on working memory, similar to that of older students. So long as numbers were small, children were able to solve the problems with accuracy. However, the larger the numbers, even if less than six, the greater the number of mistakes occurred.

Overall, researchers have been able to identify several concepts and skills that preschool and kindergarten students are able to master. The concepts are based upon Gelman and Meck's 5 counting principles: one-to-one correspondence, stable order, cardinality, order irrelevance, and abstraction (Briars & Siegler, 1984; Gelman & Meck, 1983). The number sense skills include number identification, counting, magnitude, and completion of informal addition and subtraction problems (Bertelli et al., 1998; Clarke & Shinn, 2004; Klein & Bisanz, 2000; Murray and Mayer, 1988; Resnick, 1989; Sophian, 2000; Wright, 2008). However, although this research has been completed and the literature explains the skills that preschool and kindergarten students are able to demonstrate with accuracy, there is still a need for more research within the area of early childhood mathematics development. Specifically, there has been no research to suggest a model of early mathematics development that parallels the model of early literacy. This type of research is important in order to understand mathematics development in young children. This understanding will allow educational professionals to guide specific teaching and intervention techniques for young children who struggle in acquiring mathematics skills.

## Appendix L: Author Biography

Cara Smith began her academic career in psychology at the State University of New York at Geneseo in 1998. While there, she completed research in the area of sibling and friend conflicts under the supervision of Dr. Ganie DeHart. Cara was a co-author and presenter of a poster presentation titled, “The Social Context of Children’s Conflicts and Averted Conflicts with Siblings and Friends” at the Conference on Human Development on April 6, 2002. Cara received many honors while at SUNY Geneseo including being inducted into the Phi Eta Sigma and National Order of Omega Honors Societies, and graduated cum laude from SUNY Geneseo with a Bachelor of Arts degree in May 2002.

The next step in Cara’s academic career was being accepted into the Alfred University doctoral school psychology program in August 2002. She graduated with her Master of Arts degree in School Psychology in May 2004, her Certificate of Advanced Study in School Psychology in May 2006, and her Doctor of Psychology degree in School Psychology in May 2010. While in her first year at Alfred University, Cara was inducted into the Psi Chi National Honor Society.

Since obtaining her Certificate of Advanced Study in 2006, Cara has worked in two school districts. She began her career in the Oswego City School District, where she worked as a school psychologist and local Committee on Special Education Chairperson. While there, she worked at the High School level and was co-author with Dr. Erik Laursen on the published article, “Jason and the Flaming Hamsters of Death” in the “Reclaiming Children and Youth” journal. This article was a description of the use of the Life Space Crisis Intervention technique within one of her regular counseling sessions.

In September 2007, Cara began working in the Cortland City School District as a school psychologist and local Committee on Special Education Chairperson. She spent one year at the Junior-Senior High School level, and has spent the rest of her time at Franklyn S. Barry Elementary School.

Cara now enjoys being a guest presenter at local colleges such as SUNY Oswego and SUNY Cortland. Her presentations have been made in various education courses at these colleges. The specific topics of these presentations include special education services in schools, the classification of students with disabilities, Response to Intervention, and best practices on instructing students with disabilities in the regular education classroom.