A Thesis Presented to

The Faculty of Alfred University

Pythagoras to Plato: The Intersection of Three Disciplines

by

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In partial fulfillment of

the requirements for

the Alfred University Honors Program

5/9/2022

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Preface

There are only three surviving extensive primary sources on Pythagoras' personal history known to me. These are *On the Pythagorean Way of Life* by lamblichus, *Lives of Eminent Philosophers* by Diogenes Laerteus, and *Life of Pythagoras* by Porphyry. These were all written hundreds of years after his death, well past the time of Plato. The assessment of Pythagoreanism I provide in this work is based upon the inferences of historians interpreting the work of older historians interpreting the work of untrustworthy historians who recorded their account in a dead language about a man who lived over 500 years before their time.

It is almost impossible to say anything about Pythagoras with certainty other than that he has existed in the minds of mathematicians for longer than Christ has in the minds of Christians. This fact alone, however, makes Pythagoras a subject worthy of study. It is rare for a mathematician to be as well known as he is. He is probably the only mathematician that almost every adult knows. No matter who you are, you probably had to use his theorem to solve those infamous word problems that involve an object sticking straight up, that required you to return the distance between its tip and its shadow to get a high school diploma. Trustworthy international scientific institutions are a product of the modern era, so for the people of ancient Greece certainty of knowledge did not come from such a place. Pythagoras' beliefs held influence even before the Academy was established by Plato. What the people of Pythagoras' time had was a little bit of information from ages past, orally committed to memory, then relayed between generations down to the era when it was transcribed into a relatively small number of documents, only a few of which we know the contents of. Every new dispenser of lore imparts a bit of their own bias onto the stories they tell. I am no exception, and neither are the ones I cite.

The purpose of this thesis is to compare the philosophy of the Pythagoreans and the philosophy of Plato in order to determine the extent to which Plato was influenced by Pythagoras. To do so we will be analyzing writings about Pythagoras and his followers to piece together what makes Pythagoreanism distinct, and then compare it to the philosophical beliefs we find in Plato's writings, particularly *Timaeus*, *Phaedo*, *Republic*, and *Meno*.

The thesis will be divided into three chapters so we can understand how Pythagoreanism influenced Plato's philosophy. First we need to know what we can qualify as reliable, so the first chapter will be dedicated to what direct and indirect evidence we have available to us and what they tell us about Pythagoras himself. We also need to know what Pythagoreanism is in general, so our second chapter will be a description of the relevant parts of the Pythagorean philosophy along with its evolution. Finally, the third chapter will be an analysis of Plato's writings which, using the knowledge gathered in the previous two chapters, we can distill into concepts that are Pythagorean (or Pythagorean-like); and I will argue why it is likely these concepts came from Pythagoreanism.

In this thesis, I will claim that Pythagoras himself believed and passed down to his followers the concept of metempsychosis, the transmigration of souls from one body to another after death, which Plato then adapted to his philosophy. We will also explore the divergence of Pythagoreanism into two schools of thought: Mathematical Pythagoreanism and Acousmatic Pythagoreanism. The divergence of these schools led to breakthroughs in science and political upheaval in the late Pythagoras' home region, Magna Graecia (Southern Italy), affecting the rest of the Greek world. The Mathematical Pythagoreans contributed greatly to the public discourse and ideas from their belief system made their way into some of the metaphysics, cosmology, and mathematics presented in Plato's dialogues.

I will demonstrate the veracity of these claims through careful analysis of our primary sources, one of which being Plato's student and his most important critic, Aristotle. He and his students also wrote about the Pythagoreans and criticized them for applying the same type of a priori justification to their beliefs as Plato.

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Introduction

Pythagoreanism was a philosophy that existed before the time of Socrates, founded by a Greek philosopher named Pythagoras. Those who find the name familiar often identify it with the Pythagorean Theorem from mathematics or possibly Pythagorean intervals in music. This is because Pythagoras and the followers of his philosophy made important discoveries in music and mathematics. How much of this is actually the work of Pythagoras himself as opposed to the work of his followers is unclear. In fact, little is known about Pythagoras himself since there are very few firsthand sources from the time period in which he lived (ca. 570 – ca. 490). Looking past all the legends about his life, most of our sources indicate that Pythagoras was a citizen of the Greek city state of Samos (near the coast of modern-day Turkey before fleeing his home due to the reign of a dictator name Polycrates. Most of our sources come from followers who were so far removed from the original movement, they were dubbed Neopythagoreans. Many of the things Neopythagoreans said about Pythagoras are mythological in nature. Some of the legends have him communicating with animals, living multiple lives, being in two places at once, even owning a Thracian god (Zalmoxis) as a slave.

Some of the few objective sources on Pythagoras were written by Aristotle, most of which were lost. We do still have some sources however, like his *Metaphysics* which dedicate a whole three paragraphs to the Pythagoreans. This may not sound like much, but when cross referenced with other sources, many of them fragmentary, we start to get a better idea of what the Pythagoreans were like. One important detail discussed by Aristotle, which was the subject of scholarly debate for a century, was the two "branches" of Pythagoreanism mentioned by Aristotle. These two branches were called the acousmatics and the mathematicians.

How these two branches came about relates to why Pythagoreanism started in the first place. After Pythagoras' flight, Aristotle tells us that older and younger men eventually joined Pythagoras in establishing a colony in southern Italy (a major area for Greek settlement at the time). The older men, who were more politically active, had a hard time grasping the scientific concepts that he frequently espoused, but valued his teachings. They became known as the acousmatics, because they would silently listen to him behind a curtain called the *acousmata*. The younger men listened carefully to Pythagoras' teachings and asked him questions; they would attend his demonstrations. The schism between these two schools of Pythagoreanism is said to have been because a student of Pythagoras, Hipassus of Metapontum, published a mathematical paper demonstrating how to construct a dodecahedron (a 12 sided solid with equal sides) inside a sphere. Acousmatics considered it blasphemous to publish either the works of Pythagoras or works based on his teachings since they believed the teachings must be secret. Hence the mathematicians were considered heretic Pythagoreans by the acousmatics.

The mathematicians would continue to publish works and expand on Pythagorean science. The subjects of geometry, arithmetic, astronomy, and harmonics became what is known as the Pythagorean quadrivium, a curriculum for students living in Pythagorean communities. The Pythagoreans held political power in southern Italy until the popularity of democratic movements led to an overturning of the privileged acousmatic sect who were put into exile (which is presumably why Aristotle was familiar enough with them to know about this).

Mathematical Pythagoreans contributed significantly to science and were well known around Plato's time. Two important figures which are mentioned in this thesis (besides Hipassus who was said to be the originator of the mathematician movement) are Archytas of Tarentum, a friend of Plato and famous statesman-scientist, and Philolaus of Croton, a teacher of Archytas and innovator in music theory.

In order to understand how mathematical Pythagoreanism influenced Plato, we need to define 3 terms: recollection, metempsychosis, and harmonia. Recollection is the doctrine that Plato invokes in his Socratic dialogues, *Meno* and *Phaedo*, to describe remembering information from past lives. It is an explanation for why we can learn things independent of experience. A lot of Pythagorean legends imply that Pythagoreanism espoused a more simple version of this belief which was not used to explain any kind of learning.

Metempsychosis is the concept that the soul (consciousness) of a person can exist independent from their body and move into a new body. Pythagoreans very directly espoused this doctrine, believing Pythagoras' soul to have inhabited many different bodies. Plato affirms the belief that souls can be independent from the body in *Phaedo* but does not directly affirm the cycle of life and death that would accompany reincarnation (the movement of a soul into a new body). Instead, Plato considers the idea but does not say whether or not it's true. Nonetheless, this still indicates he is being influenced by the Pythagorean notion of metempsychosis.

Harmonia is a term coined by Philolaus of Croton to describe the universal significance of musical ratios. He would later use this to justify the idea that the planets are spaced according to harmonic ratios and emit frequencies that we cannot hear. This appears in both Plato's *Timaeus* and *Republic*. In *Timaeus* Plato uses this idea to explain how the divine creator formed the soul of the world. In his *Republic*, he references Philolaus by claiming that the sciences of astronomy and harmonics are "kindred."

Overall Plato was influenced by Pythagoreanism in very clear ways. Although we do not possess much first-hand information about who Pythagoras was or the specifics of his philosophy, we can easily identify what beliefs originated with him. Metempsychosis is found in the *Meno* and *Phaedo*. Harmonia is a recurring theme in *Timaeus* and is mentioned in Plato's *Republic* along with Pythagorean texts. This, with Plato's close friendship with a Pythagorean and apparent admiration for pythagoreans (since they are a recurring type of character in his work), justifies the notion that Plato was influenced by Pythagoreanism.

Although we do not know much about Pythagoras, his philosophy and the discoveries of his followers still influence us today. The idea that something could be proven abstractly and then applied generally is the cornerstone of modern mathematics. The Pythagorean interval and the numerous innovation s of Philolaus and Archytas are still foundational elements of music theory. The idea that the soul can exist separate from the body is still a belief in many mainstream religions. Plato remains a major figure in western philosophy and the aspects of Pythagoreanism that he utilized in the formation of Platonism continue to influence the world.

Chapter I: The Story of Pythagoras

Section 1.1: The Pythagorean Legend

There were many myths told about Pythagoras which elevated him to (and sometimes above) divine status. One of the most common is that he possessed bilocation, the ability to be in two places at once. He was also rumored to have a "golden thigh." He was even supposedly able to talk to animals, successfully convincing a bull to abstain from eating beans and persuading a wild bear to adopt pacifism.¹

Many of these myths are present in our primary source reports (Laertius, lamblichus, and Porphyry) of Pythagoras' life and they give us a very flawed understanding of who Pythagoras was since they are laden with mysticism and contradictions. Fortunately, for about a century, scholars have been laboring to give us an accurate picture of who Pythagoras actually was.

Much of what is written about Pythagoras by Porphyry is Platonic in nature. For instance, Porphyry said Pythagoras taught his followers to turn away from corporeal things and focus on the eternal infinite things and that you can free your soul through contemplation of mathematics. This mirrors ideas put forward by Socrates in *Phaedo.*² One conclusion we could draw from this is that Plato was inspired by Pythagoras who initially conceived of this idea, but there is little textual evidence to support that. The

¹ Charles H. Kahn, *Pythagoras and the Pythagoreans: A brief history* (Indianapolis, IN: Hackett Publishing Company Inc, 2001), 5.

² Christoph Riedwig, *Pythagoras: His Life, Teachings, and Influence* (Cornell University Press, 2012), 21.

more likely explanation is that Porphyry was influenced by Plato since he was a Platonist.³ This is still significant however, because the affiliation of Platonic ideas with Pythagorean ones appears to be a natural consequence of their similarities.

The same can be said of lamblichus who frequently projected Platonic and Socratic beliefs onto Pythagoras. According to lamblichus the Pythagoreans "deprecated those who peddle learning... and, in short, make profit out of gymnasia and youths, demanding a wage for things on which no price should be put."⁴ The language used here is nearly identical to the language used in the Platonic dialogue *Sophist* to criticize the same actions. Since lamblichus was a known Neoplatonist,⁵ we need to take his assigning of Platonist concepts to Pythagoreanism with appropriate hesitance.

Nonetheless, to truly discern Pythagorean ideas from Platonic ones, we have to analyze sources which predate Plato, sources that no longer exist in extant form. However, we have fragments of these sources preserved through citation of older texts in our primary sources. So, while some passages may be misleading due to selection bias, we can at least learn some things about the authentic Pythagorean philosophy if we apply an appropriate amount of skepticism to them.

³ Riedwig, *Pythagoras*, 22

⁴ lamblichus, On the Pythagorean Way of Life, 239 (245).

⁵ He even famously preserved portions of the dialogue *Sophist*.

Section 1.2: Indirect Sources

Diogenes Laertius is generally uninformed and indifferent to the philosophies of people he cites, but the fragments he cites are very useful.⁶ For example, returning to metempsychosis, the oldest work we know of mentioning Pythagoras is a satirical poem written by one of his contemporaries, Xenophanes of Colophon, and it mocked Pythagoras for a belief he had about the transmigration of souls. However, the only surviving fragment of it is cited in Laertius,

They say that, passing a belabored whelp, He, full of pity, spake these words of dole: 'Stay, smite not! 'Tis a friend, a human soul; I knew him straight whenas I heard him yelp!'⁷

From this we can infer that Xenophanes knew of Pythagoras and believed it characteristic of him to advocate for metempsychosis.⁸ Zhmud further argues "It is clear from Xenophanes' words that metempsychosis was already widely known at the turn of the sixth and fifth centuries in Magna Graecia and was associated with the name of Pythagoras."⁹ So even during the time of Plato, we can say that metempsychosis was considered a distinctly Pythagorean doctrine.

⁶ Herbert S. Long, Introduction to *Lives of Eminent Philosophers* ed. Jeffery Henderson (Cambridge, Massachusetts: Harvard University Press, 1972), xvii.

⁷ Diogenes Laertius, *Lives of Eminent Philosophers*, trans. R.D. Hicks (Cambridge, Massachusetts: Harvard University Press, 1972), 353.

⁸ Leonid Zhmud, *Pythagoras and the Early Pythagoreans* (Oxford University Press, 2012), 31.

⁹ Zhmud, *Pythagoras*, 31.

Conveniently Pythagoras and Heraclitus' lives overlapped as well¹⁰ and Laertius' work also contains a fragment from Heraclitus, "Pythagoras son of Mnesarchus pursued inquiry beyond all other men, and in this selection of his writings, made himself a wisdom of his own, showing much learning, but poor workmanship."¹¹

One might take this as an indication that there are more sources, written by Pythagoras himself, that exist. But, according to Zhmud "In reality Heraclitus is speaking of the use of someone else's books, not of writing his own."¹² What works Heraclitus meant, however, has been the subject of heated debate and flagrant speculation. Close linguistic analysis gives the interpretation that Heraclitus is essentially claiming Pythagoras recklessly copied other people's ideas to suit his own needs.¹³

This implies Pythagoras certainly absorbed some elements from various philosophies conceived before, or during, his time and likely tried to tie some of them together into his own. The language used, within the context of the "background of the early tradition," indicates as much since it concedes he is very knowledgeable but charges him with misusing that knowledge.¹⁴

¹⁰ W.K.C. Guthrie, A History of Greek Philosophy (Cambridge, Cambridge University Press, 1967), 157.

¹¹ Laertius, *Lives*, 325.

¹² Zhmud, *Pythagoras*, 33.

¹³ Zhmud, *Pythagoras*, 34-35.

¹⁴ Zhmud, *Pythagoras*, 35.

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Zhmud also tells us that "judging by Heraclitus' reaction, the reputation of Pythagoras had extended well beyond the boundaries of Magna Graecia by the first quarter of the fifth century."¹⁵ It is not a great leap to suggest that the population of Athens had been exposed to Pythagoreanism well before Plato's period of activity in the early half of the 4th century BC.¹⁶ Heraclitus makes use of musical proportion and harmony, which are widely accepted to be ideas that originated among the early Pythagoreans, in his own philosophy and he lived in Ephesus,¹⁷ which is farther from the Pythagorean center of Croton and later Metapontum than it is from Athens, one of the foremost centers of trade at the time. If Pythagorean ideas in Plato's work.

We also find the influence of early Pythagoreanism in the philosopher Democritus. In addition to the book he wrote called *Pythagoras* that is now lost to time, we know that Democritus studied with the Pythagoreans thanks to the testimony of his contemporary, Glaucus of Rhegium. The nature of early Pythagoreanism is contested within the scholarly community since some scholars claim it was more focused on mythology and esotericism in the beginning and later evolved into something more grounded in Mathematical principles. Others argue that Democritus' admiration for Pythagoreanism must have stemmed from an appreciation for its application of mathematics since he himself was a mathematician and spent a significant amount of time studying among the Pythgoreans.¹⁸ I side with the latter argument since it is a

¹⁵ Zhmud, Pythagoras, 34.

¹⁶ We know that Plato lived between the years 428 BC - 347 BC until he died at around the age of 80.

¹⁷ Zhmud, *Pythagoras*, 34.

¹⁸ Zhmud, *Pythagoras*, 45.

common misconception that mysticism is always absent from scientific endeavors. This is especially incorrect as it applies to the past. I think in this time period it is sufficiently likely that the two co-mingled enough for Pythagoreanism to exist as both a religious and scientific movement.



Figure 1. Map of ancient Greek settlements in

Magna Graecia

Source: Wikimedia Foundation

Figure 2. Map of ancient Greek settlements in

Ionia

Source: Wikimedia Foundation

Chapter II: Pythagoreanism's Evolution

Section 2.1 - The Pythagorean Schism

The initial Pythagorean communities in Southern Italy were established after Pythagoras fled from the island of Samos in Ionia when a dictator named Polycrates took control. According to Aristotle, the political elite of certain city states accompanied Pythagoras and he addressed the older of these men in "simple style" to avoid confusion when he was first establishing his communities in Italy where they enjoyed a great deal of political influence. Evidently these men were so caught up in political matters that they had no time to understand the sciences that Pythagoras would teach his followers.¹⁹

Further, there were two groups of Pythagoreans, which we call the acousmatics and the mathematicians, that diverged from one another, "of these two, the acousmatics were recognized to be Pythagoreans by the others [the mathematicians], but they did not recognize the mathematicians [as Pythagoreans]."²⁰ The reason for this divergence has to do with the differing foci of their philosophical inquiry.²¹

The acousmatics were the older men who Pythagoras addressed in "simple style," their system of beliefs were based not on geometry or science but on apparent

¹⁹ Philip Sydney Horky, *Plato and Pythagoreanism* (Oxford University Press, 2013) 56.

²⁰ Horky, *Plato and Pythagoreanism*, 14.

²¹ Horky refers to this concept using the Greek word *pragmateia*.

"facts" which they observed.²² They were said to possess knowledge of "what one is to do," presumably with regards to leading a fulfilling life, rather than "reason why they are to do."²³ They were named after the *acousmata*, a curtain behind which students of Pythagoreanism would silently listen to their master's lectures.

The Mathematicians were the younger of these men and Aristotle implies that they took what Pythagoras said with almost the same degree of seriousness as the acousmatics, but the acousmatics did not see them as real Pythagoreans. Instead, the Acousmatics referred to them as "so-called" Pythagoreans.²⁴ The Mathematicians were known to engage in public demonstrations of Pythagorean principles. According to Aristotle, they engaged in two types of demonstration:

- That all things come from number since they "possess the attributes of number."
- 2. That "spacial magnitude" arises from their axioms, the "objects of mathematics."²⁵

Both groups agreed that the cosmos was ordered by abstract metaphysical principles that were removed from the physical world, and that political structures ought to run parallel to the order of the cosmos. Acousmatics and Mathematicians shared roughly the same set of ethical imperatives and rituals, which were orally passed down from Pythagoras during the first five years of a student's education (the key difference

²² Horky, *Plato and Pythagoreanism*, 37.

²³ Horky, *Plato and Pythagoreanism*, 17.

²⁴ Horky, *Plato and Pythagoreanism*, 30.

²⁵ Horky, *Plato and Pythagoreanism*, 21.

was that the mathematicians felt they were justified in sharing, questioning, and expanding on Pythagorean concepts).

The difference between the two groups so far as Aristotle was concerned, can be categorized by two important questions and how each of the two groups answered them: "what is?" and "what is to the greatest degree?" According to Horky, these questions "formed the background for Plato's dialectical response to Pythagoreanism."²⁶ Since Acousmatics focused primarily on a third, less pertinent, question "what is to be done?"²⁷ I infer that Mathematical Pythagoreans made a greater impact on Plato (since they made an effort to propagate their beliefs to the general public) and are thus more relevant to our topic. Additionally, Aristotle mainly concerned himself with the Pythagorean pursuits that fall within the realm of the first two questions, thus we have more evidence that can be analyzed in this realm.

To better explain the two questions, "what is?" is an attempt to explain the underlying substance of all that exists in the universe. And "what is to the greatest degree?" ponders the order of these underlying substances (in other words, how they rank relative to one another). Both of these are questions in metaphysics, the branch of philosophy concerned with the fundamental nature of reality. The Acousmatics sought to use Pythagoras' teachings as a guide for how to live everyday life; they did not feel the need to question things that they could not understand. They took the knowledge

²⁶ Horky, *Plato and Pythagoreanism*, 37.

²⁷ Horky, *Plato and Pythagoreanism*, 9.

Pythagoras passed down for granted. Hence, they used Pythagoreanism as a way to *decide* what needed to be done. On the other hand, the Mathematicians asked questions and gave public demonstrations, expanding on Pythagoras' work. This means they used Pythagoreanism as a methodology, a way to *learn* what needed to be done by describing the fundamental substances of the universe.

The Acousmatics claimed that the rationale of the mathematicians was not derived "from Pythagoras, but rather from Hippasus"²⁸ who, legend has it, was condemned to drown at sea for his blasphemy.²⁹ According to lamblichus, he was drowned at sea for "offending the gods" by publishing demonstrations of how to construct a dodecahedron inside a sphere.^{30 31} The justification given was that he disclosed a secret discovered by Pythagoras himself. This tells us that the Acousmatics were more secretive about Pythagorean doctrines as opposed to the Mathematicians, who allegedly follow a tradition stemming from Hippasus of publishing it.

Horky suggests that a passage on Hippasus cited by Diogenes Laertius actually derives from Theophrastus of Eresus, one of Aristotle's students.³² Theophrastus implies that Hippasus and Herodotus shared the beliefs that "the universe is unified, continually in motion, and limited, and that fire, the first principle of the universe, is also

²⁸ Horky, *Plato and Pythagoras*, 56.

²⁹ It is a very common misconception that Hippasus was murdered at sea for the discovery that $\sqrt{2}$ is irrational. For the more mathematically inclined reading this, look to Gregory's book (31-32) for a more detailed explanation.

³⁰ lamblichus, On the Pythagorean, 241 (247).

³¹ Horky, *Plato and Pythagoras*, 16.

³² Horky, *Plato and Pythagoras*, 63.

unified, in motion, and limited."³³ Further, Theophrastus references Plato together with the Pythagoreans and claims that they made "all things desire to imitate fully the first principles."³⁴ This aligns with the Aristotelian criticism that the Mathematical (or "so-called") Pythagoreans frequently sought first principle *a priori* explanations for observable phenomena.

These examples reinforce the claim that Hippasus indeed employed methodology characteristic of Mathematical Pythagoreans and that those same methodologies are being considered alongside Plato's by Aristotle in *Metaphysics*. This draws a connection between the philosophy of Plato and the philosophy of Hippasus, the "so-called" father of Mathematical Pythagoreanism.³⁵

³³ Horky, *Plato and Pythagoras*, 66.

³⁴ Horky, *Plato and Pythagoras*, 67.

³⁵ In addition to a philosophical schism there was also an ideological schism between the democratizing ideals of the mathematicians and the more elitist acousmatics which is not as important to this thesis. See Chapter 3 of Horky for more information.

Section 2.2 - Philosophy of the Pythagoreans

Aristotle indicates in his *Metaphysics,* that Mathematical Pythagoreans believed non-perceptible mathematical objects (i.e. anything that may be formally defined and used in proofs) were first principles.³⁶ In addition, he says they also considered natural phenomena with motion to be the focus of their philosophical inquiry. Aristotle writes

In the times of these men [i.e. Leucippus and Democritus] and before them, the "so-called" Pythagoreans, were the first to latch onto mathematics. They advanced mathematics and, by being brought up in it, they began to believe that the principles of mathematics were the principles of all things in existence. And since numbers are first among these [i.e. beings] by nature, they seemed to see many resemblances in things that are and things that come into being, rather than in fire, earth, or water.³⁷

The fundamental principle of mathematical proofs is that by proving something is true with general assumptions, we can ensure that conclusions of the proof will follow, provided that the assumed conditions are met. These "so-called" or, as Horky has sufficiently demonstrated, mathematical Pythagoreans, who may have been among the first to use mathematical proofs, believed that the abstract objects used in proofs, numbers, must be a fundamental building block of all reality since they can be used to

³⁶ Meaning they reside in the highest part of reality, they are a first cause.

³⁷ Horky, *Plato and Pythagoras*, 22.

establish general relations between real things. Thus they concluded, number is the underlying essence of all things, a first principle.

The Pythagoreans also believed that each number corresponded to an abstract concept in ascending order such as truth, justice, and opinion.³⁸ In other words, for every object in the universe, there exists a number such that said number is an attribute of that object. Aristotle took issue with this notion because it doesn't allow for motion or change, which the Pythagoreans claimed to have as the focus of their philosophical inquiry, since it ascribes numbers as individual unchanging entities which exist in their own right, permanently representing subjective attributes.³⁹ If something is changing at a constant rate, then is it also transforming its virtue? Pythagoreans likely would have made this an exception in their doctrine of numerology.

Another of Aristotle's criticisms was that the Pythagoreans assigned physical and metaphysical relations that have no apparent merit using this same a priori reasoning from the assumption that "all is number". The particular assertion he criticized was that, in addition to the 5 observable planets (Mercury, Venus, Mars, Jupiter, and Saturn), the sun, the moon, the earth, and the stars; there had to be a counter earth in order to bring the number of bodies up to ten, which they asserted all rotated around a central flame, obscured by the counter earth⁴⁰. One salient objection to this is that there doesn't seem to be a good reason to group all the stars into one celestial body. The Pythagoreans

³⁸ This bias for ascending order could be more of a reflection of Aristotle's cosmology than

Pythagoreanism since, to him, things took their position in the heavens based on how high up they went. ³⁹ Horky, *Plato and Pythagoras*, 20.

⁴⁰ Gregory, "Number and Numerology", 38.

went so far as to invent a non-observable counter-earth just to obscure the central flame. One may wonder, why expend such intellectual effort on something so unlikely? The answer lies in their numerology which was based on their first principle.

Limited	Unlimited
Odd	Even
One	Plurality
Right	Left
Male	Female
Rest	Motion
Straight	Curved
Light	Darkness
Good	Evil
Square	Oblong

There are ten dichotomies that Pythagoreans held to be first principles:



Ten is a very prominent number in Pythagorean numerology, and the reason why relates to their notorious belief in the importance of harmonic ratios. The Pythagoreans held the sum of the first four integers in high esteem because those integers are present in the musical intervals that correspond to the octave, the perfect fourth, and the perfect

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fifth (2:1, 3:2, and 4:3 respectively).⁴¹ The Pythagoreans were among the earliest innovators in music theory due to their discovery that mathematical ratios affect the tone of sounds that resonate from objects. This numerology is also utilized in a common Pythagorean symbol called the Tetraktys (see above in figure 1), ten points arranged in vertically descending symmetrical columns.⁴² This discovery was essential to composing melodies since without it there can be no musical scale.

Mathematical Pythagoreans of the following years, between Pythagoras and Plato, would further expand on the music theory and mathematics of Pythagoreanism. The two most important figures of this movement for us are Archytas of Tarentum and Philolaus of Croton. They are two of the handful of Pythagoreans in the timeframe we're studying who are known to have composed significant works. While none of these works are left in one piece, there are fragments which come from quotations of their work in other sources. The next chapter will be a step by step analysis of the philosophies of these individuals brought up as they are relevant to Pythagorean concepts that we find in Plato's dialogues.

⁴¹ Kahn, *Pythagoras and the Pythagoreans*, 25.

⁴² Andrew Gregory, "The Pythagoreans: Number and Numerology" in *Mathematicians and Their Gods: Interactions Between Mathematics and Religious Beliefs*, 28.

Chapter III: To Plato

The presence of transmigration of souls in Plato's dialogues indicates some Pythagorean influence. As we have established, the transmigration of souls is a doctrine that was a distinct feature of Pythagoreanism. It can be found mentioned throughout Plato's *Meno* and *Phaedo*. It is worth noting that Pythagoreanism was not the only presocratic philosophy that incorporated metempsychosis, Orphism had a similar concept. The important difference however, is that Orphism preached salvation from the suffering caused by this cycle through adherence to its beliefs.

Pythagoreanism didn't categorize the cycle of death and rebirth as a form of suffering and did much more to emphasize the positive metaphysical ramifications of the soul's independence from the body and remembering past lives (as we see the Neopythagorean authors doing when they recount legends about Pythagoras).⁴³ I would argue that the metempsychosis we find in Plato's dialogues is more akin to the metempsychosis of Pythagoreanism due to its emphasis on those two things.

⁴³ Zhmud, *Pythagoras*, 232-233.

Section 3.1 - Meno

One of the core themes of the *Meno* is the doctrine of recollection, that learning is just the recollection of a priori knowledge. Plato depicts this when he presents a conversation between Socrates and a slave boy (attendant) of the eponymous Meno. After Socrates spends a considerable amount of time asking the boy questions, leading to him having a greater grasp of geometry, Socrates suggests that there must have been a time beforehand where the boy acquired this knowledge. Since he was raised in Meno's house, and Meno knows he was never taught geometry, they agree it must have been before the boy was born (before he was a human being). Hence, Socrates asks,

Socrates: So if in both of these periods—when he was and was not a human being—he has had true opinions in him which have only to be awakened by questioning to become knowledge, his soul must have had this cognizance throughout all time? For clearly he has always either been or not been a human being.

Meno: Evidently.

Socrates: And if the truth of all things that are is always in our soul, then the soul must be immortal; so that you should take heart and, whatever you do not

happen to know at present—that is, what you do not remember—you must endeavor to search out and recollect?

Meno: What you say commends itself to me, Socrates, I know not how.⁴⁴

While many easily identify the doctrine of recollection as a Platonic idea, it is also a Pythagorean concept. Kahn writes "For the Pythagoreans recollection meant, first of all, remembering one's previous incarnations (as Pythagoras himself was reported to have done)."⁴⁵ The legends that circulated about Pythagoras depicting him as a wayward soul who carries knowledge between lives is found here in Plato's work where it has been generalized with a more philosophical framework and removed from the aggrandizing mysticism of Pythagoreanism. It is interesting that Plato felt the need to specify that the knowledge was learned before the subject was a "human being." This could relate to the Pythagorean belief that souls could be reborn as animals, not just humans. It is also worth noting that within the text, Socrates seems to be conflicted on whether this hypothesis about the soul is true or not, which is likely a reflection of Plato's thoughts as well.

Furthermore, the doctrine of metempsychosis is explicitly mentioned in the *Meno*. Plato (through Socrates) references priests and priestesses "who have studied so as to be able to give a reasoned account of their ministry... They say that the soul of man is

⁴⁴ Plato, "Meno," in *Plato: Complete Works*, ed. John M. Cooper (Indianapolis: Hackett Publishing Company, 1997), 886 (86 a-b).

⁴⁵ Kahn, *Pythagoras and the Pythagoreans*, 51.

immortal, and at one time comes to an end, which is called dying, and at another is born again, but never perishes...^{*46} Here we see that Plato is giving an account of the immortality of the soul and its rebirth after death. This suggests that Plato was modifying these previously "magical, ritualistic" ideas into a more organized epistemic philosophy predicated on a priori knowledge,⁴⁷ which is another similarity we find between Platonism and Pythagoreanism.

⁴⁶ Plato, *Meno*, 880 (81a-b).

⁴⁷ Kahn, Pythagoras and the Pythagoreans, 51.

Section 3.2 - Phaedo

The topic of Plato's *Phaedo* concerns the fate of the soul after death, and the ideas Plato expresses here once again bear striking similarities to the Pythagorean transmigration of souls. It is no coincidence that the two interlocutors, Simmias and Cebes, are confirmed to be Pythagoreans. It could be argued that Plato was trying to draw a connection here between Pythagoreanism and Socrates,⁴⁸ "How is this Cebes? Have you and Simmias, who are pupils of Philolaus, not heard about such things?"⁴⁹ Philolaus, as we have discussed, is undoubtedly a Pythagorean. The fact that Socrates is presented as being familiar with Philolaus' works and takes knowledge of them for granted, implies that Plato, the writer of this dialogue, feels the same way.

Later in the dialogue, Socrates presents a view of what happens to the soul after death, which Simmias agrees with,

We believe, do we not, that death is the separation of the soul from the body, and that the state of being dead is the state in which the body is separated from the soul and exists alone by itself and the soul is separated from the body and exists alone by itself? Is death anything other than this?⁵⁰

Thus Plato (through Socrates) is affirming that souls leave the body after death. This alone is not an example of Plato claiming that the soul of a deceased person will

⁴⁸ Kahn, *Pythagoras and the Pythagoreans*, 49.

⁴⁹ Plato, "Phaedo," in *Plato: Complete Works*, ed. John M. Cooper (Indianapolis: Hackett Publishing Company, 1997), 53 (61d).

⁵⁰ Plato, *Phaedo*, 56 (64c).

transmigrate, just that the soul is independent from the body. Thus, this alone is not inherently Pythagorean, but later Plato expands on this idea,

So by this method also we reach the conclusion that the living are generated from the dead, just as much as the dead from the living; and since this is the case, it seems to me to be a sufficient proof that the souls of the dead exist somewhere, whence they come back to life.⁵¹

This is effectively a justification for the Pythagorean view of souls, as entities which are not disposed of after death but instead go elsewhere, fueling the cycle of life: "the living are generated from the dead." The main difference between the Platonic and Pythagorean conception of the soul is in *where* the soul may go after death, something about which Socrates seems uncertain. The Pythagoreans would have said that the soul transplants into a new body after death, and by the arguments presented that seems to be what is implied. But again, Plato does not seem wholly convinced.

Even though it is not outright stated that he believes in something identical to the metempsychosis of Pythagoreanism, the main question in *Phaedo* is clearly derived from, and prompts an analysis of, specifically the Pythagorean view of the soul. Socrates' cyclical argument (the argument we just discussed) seeks to answer the question of what happens after death, in dialogue with two Pythagoreans Simmias and

⁵¹ Plato, *Phaedo*, 62-63 (72a).

Cebes, using a form of quasi-metempsychosis with fewer assumptions and a more generalizable philosophical framework.

One way Plato and Pythagoras' philosophies appear to differ is in the specific way they construct the cosmos. While both use a type of mathematics in their constructions, Pythagoras is mainly concerned with a broadly defined arithmetical construction of the cosmos (out of number) and Plato presents a more specific geometric construction. Gregory writes

"For Plato there were two fundamental triangles, which formed either a more complex triangle or a square, which in turn formed three dimensional shapes: tetrahedron, octahedron or icosahedron from the complex triangles or a cube from the squares. These shapes were fire, air, water, and earth respectively."⁵²



Figure 4. Tetrahedron, Octahedron, Cube, and Dodecahedron Source: Wikimedia Foundation (modified by combining the images together).

While there are indeed many differences between them, the overall similarities between Platonic and Pythagorean cosmology in their use of mathematical objects as first

⁵² Gregory, *Number and Numerology*, 29.

principles attests to a Pythagorean influence in Plato's philosophy and prompts further investigation.

Section 3.3 - Timaeus

Over the course of *Timaeus*, Plato utilizes Pythagorean mathematical concepts to describe how the world-soul was constructed . For Plato, the world-soul had to be constructed by a supreme craftsman, very similar to the concept of intelligent design in Christian theology. He constructed this world-soul by putting together three elements (each one having two "varieties": divisible and indivisible): Sameness, Difference, and Being. The three of these were formed into divisible compound substances. While this may sound strange, the reader must remember Plato believed there was a higher order of reality that consisted purely of ideas (the world of forms). The craftsman then lays the mixture out as a single length which he chops pieces off of.⁵³ Here Plato utilizes "mathematical Pythagorean approaches to music theory" to construct the universe "in accordance with the principles of symmetry and concordance."⁵⁴

When the craftsman divides the world soul, he takes "the intervals in the series of the powers of 2 and the intervals in the series of powers of 3" and Plato writes

The insertion of these links formed fresh intervals in the former intervals, that is to say, intervals of 3:2 and 4:3 and 9:8, He went on to fill up the 4:3 intervals with 9:8 intervals. This still left over in each case a fraction, which is represented by

⁵³ Plato, "Timaeus," in *Plato: Complete Works*, ed. John M. Cooper (Indianapolis: Hackett Publishing Company, 1997), 1239 (35-36a).

⁵⁴ Horky, *Plato and Pythagoreanism*, 255.

the terms of the numerical ratio 256:243. And thus the mixture, from which He had been cutting these portions off, was now all spent.⁵⁵

To the average reader this may not appear to follow a precise algorithm, but earlier in the passage Plato uses the word "mean" to describe the function from which the inputs coming from elements of these two sets (whose elements comprise the series' of powers 2 and 3) were applied to give us the ratios as outputs. These outputs are all Pythagorean Intervals. A Pythagorean interval is any musical interval with a frequency ratio equal to a power of 2 over a power of 3 or a power of 3 over a power of 2. The simplest example of a Pythagorean Interval which illustrates this is 3:2. So as we can see 3:2, 4:3, 9:8 all the way up to 256:243 are all meant to be Pythagorean Intervals, since they are the Harmonic mean of two elements, each from a series of powers of 2 and a series of powers of 3 respectively.

The Harmonic mean, which Plato used to get these Pythagorean Intervals, is one of the three Pythagorean means.⁵⁶ As the name implies, these means were used to calculate harmonic ratios for music, hence the presence of the perfect third and perfect fourth ratios among various others. It is well known that these means were originally studied by the Pythagoreans, who, as we discussed in chapter 2, considered them to be significant to the cosmology of the universe.

⁵⁵ Plato, *Timaeus*, 1239 (36a-b).

⁵⁶ The other two being Arithmetic and Geometric means.

One of the earliest sources we have who discussed the Harmonic mean was Plato's dear friend and a Pythagorean, Archytas of Tarentum. Timaeus, the main speaker of this dialogue, is most likely based on him due to their similarities (they're both statesman-scientists from southern Italy). Plato's debt to Pythagorean mathematics is no better attested than in his friendship with Archytas.⁵⁷ Plato encouraged people to study solid (in addition to planar) geometry in imitation of Archytas, who he held up as an exemplary Pythagorean.⁵⁸ Archytas clearly had a significant impact on Plato's philosophy, since Plato bases the ideas covered in the *Timaeus* on his friend's Pythagorean beliefs.

Using reasoning derived from the assumption that abstract mathematical objects are a first principle, Plato utilized a Pythagorean concept known as *harmonia* to give the spacing between celestial bodies. This concept of *harmonia* is derived from the work of a Pythagorean named Philolaus of Croton (who Simmias and Cebes were said to be students of). Philolaus studied the mathematics in musical harmony, the discovery of which is attributed to Pythagoras after which the Pythagorean Scale in Music Theory is named.⁵⁹ For Philolaus, *harmonia* was an a priori principle that was meant "to produce unity out of multiplicity by bringing diverse and discordant elements into an agreement with one another." In other words, he believed that *harmonia* was a natural consequence of those irreconcilable dichotomies which Pythagoreans held to be first principles, allowing those dichotomies to coexist with one another. The form of

⁵⁷ Kahn, *Pythagoras and the Pythagoreans*, 39-40.

 ⁵⁸ Plato indicates that Archytas was one of the first Greek thinkers to study solid geometry, being the earliest to solve the "Delian problem" of doubling the cube and doing so in three dimensions.
⁵⁹ Gregory, *Number and Numerology*, 35-37

harmonia is numerical in nature since, as Philolaus says, "all things which are known have number; for nothing can be known or understood without number," he thus defines *harmonia* as the 2:1 ratio, the octave.⁶⁰

In fact, the *Timaeus* was so similar to Philolaus' book on natural philosophy that Plato was accused of plagiarism. To fuel these accusations, he was rumored to have spent a lot of money while in Syracuse to purchase the recently deceased Philolaus' book. Based on this, the satirist Timon of Phlius mocked Plato,

You too, Plato! For you too were seized by the desire to learn from a teacher; for a great deal of money you bought yourself a little book; by it you were taught how to "write Timaeus."⁶¹

The fact that his ideas were so affiliated with a piece of Pythagorean literature and displayed cosmology derivative of Pythagorean concepts indicates that the Pythagorean influence on *Timaeus* was so strong that it was difficult for people to distinguish between it and a genuine Pythagorean work, like Philolaus' book.

⁶⁰ Kahn, *Pythagoras and the Pythagoreans*, 25.

⁶¹ Riedwig, *Pythagoras*, 117.

Section 3.4 - Republic

Pythagoreanism can lastly be found in Plato's *Republic*. In chapter VII Plato suggests a math curriculum for his ideal "philosopher king" based on the Pythagorean quadrivium, a standardized system of education that originated with the Pythagoreans, (arithmetic, geometry, astronomy, and music)⁶²:

to enter upon that study of calculation and take hold of it, not as amateurs, but to follow it up until they attain to the contemplation of the nature of number, by pure thought, not for the purpose of buying and selling.⁶³

Here we have an example of Arithmetic. It is interesting that Plato considered a theoretical knowledge of math to be a form of enlightenment above the application of math. Fixation on the nature of numbers as abstract concepts being the key to understanding everything was a core tenant of Pythagoreanism, and together with his other curricula we continue to fill out the Pythagorean quadrivium,

for geometry is the knowledge of the eternally existent.' 'Then, my good friend, it would tend to draw the soul to truth, and would be productive of a philosophic

⁶² Kahn, *Pythagoras and the Pythagoreans*, 40.

⁶³ Plato, "Republic," in *Plato: Complete Works*, ed. John M. Cooper (Indianapolis: Hackett Publishing Company, 1997), 1142 (525c).

attitude of mind, directing upward the faculties that now wrongly are turned earthward.⁶⁴

Without doubt, Plato finds geometry to be an important subject for his curriculum. He also later discusses the importance of solid geometry and laments the fact that people ignore it (yet another sign of Archytas' influence). After this, Timaeus suggests that in addition to geometry, astronomy should be a subject. While his interlocutor initially accepts, he then backpedals. Glaucon doesn't see a value in the study of astronomy, he just sees it as a way of effectively appreciating pretty pictures on the ceiling of the world. Timaeus reassures him that in fact, he doesn't mean that kind of astronomy. The type he advocates is very much related to geometry, regarding the movement of celestial bodies,

as in the study of geometry, that we will pursue astronomy too, and we will let be the things in the heavens, if we are to have a part in the true science of astronomy and so convert to right use from uselessness that natural indwelling intelligence of the soul.⁶⁵

Next Timaeus suggests adding Harmonics to the curriculum. While Glaucon, once again, initially accepts Timaeus' proposition, he finds reason to disagree remembering how musicians "waste" a lot of time tuning their instruments. Timaeus assures him that

⁶⁴ Plato, *Republic*, 1143 (527b).

⁶⁵ Plato, *Timaeus*, 1146 (530b-c).

he isn't talking about those types of musicians. Instead he draws a familiar sounding connection between music and astronomy,

'We may venture to suppose,' I said, 'that as the eyes are framed for astronomy so the ears are framed, for the movements of harmony; and these are in some sort kindred sciences, as the Pythagoreans affirm and we admit, do we not, Glaucon?' 'We do,' he said.⁶⁶

It appears that both speakers are in agreement that the combination of musical harmony and astronomy advocated by the Pythagoreans is justified, as we have seen Plato affirm once again in *Timaeus*. This time the Pythagoreans are directly mentioned as a source of inspiration for this idea. Clearly this idea of kinship between harmonics and astronomy was influenced by Pythagoreanism.

In fact, Plato's comparison between astronomy and harmonics as "kindred sciences" also uses strikingly similar language to a passage in Archytas' lost text *Theory of Harmony*:

Concerning the speed and risings and settings of the heavenly bodies they have handed down to us clear knowledge, concerning geometry and numbers, and not least concerning music. For these studies seem to be akin.⁶⁷

⁶⁶ Plato, *Timaeus*, 1146 (530d).

⁶⁷ Kahn, Pythagoras and the Pythagoreans, 44-45.

So once again it seems Plato was influenced by his friend in applying Pythagorean values (embodied in the quadrivium) to education using language which conveys the same metaphor (the word rendered as akin may also be translated as sister) for the astronomical significance of music.

It is no surprise that Plato was influenced by Pythagoreanism, and the prevalence of Pythagorean ideas such as metempsychosis, recollection, *harmonia*, and the conflation of astronomy and harmonics found in his work shows it. Pythagorean metaphysics and mathematics are found littered across his dialogues, and some of his closest friends (and significant speakers in his dialogues) are well attested Pythagoreans. Platonic and Pythagorean philosophy continued to influence each other well after Plato due to their similarities; so much so, that it has become difficult for historians to distinguish between them. Therefore it is clear that Pythagoreanism underlies a large part of Plato's philosophical system, particularly that which is present in the dialogues discussed.

Pythagoreanism, although shrouded by the mists of time, continues to be influential in our society to this day. Their innovations in music theory, and the religious devotion that they had to its study, led to the discovery that different harmonic ratios produce different tones. The idea that an abstract mathematical concept can influence the physical world was a revolutionary turning point in the history of mathematics, it is the birth of mathematical proofs. Although Pythagorean proofs were rather crude "demonstrations," often supplemented with faulty logic, the concept that something

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ought to be proven abstractly and can then have its meaning extended to the properties of all relevant physical phenomena is a cornerstone of modern science. The belief that a priori knowledge allows us to learn things independent of observation has echoed throughout the centuries in western philosophy and likely has its roots, not in Plato who formalized it, but more in Pythagoras whose followers conceived of it. Finally, the doctrine of the soul as an entity that can exist independent from the body, which is carried on by a majority of the myriad of branches in Christianity and Islam, originated not from Plato, but from Pythagoras. Despite the enigma surrounding him, it is apparent that many of the things that we know and ponder in the present which were formalized by Plato, started out with Pythagoras and the Pythagoreans.

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