A Thesis Presented to

The Faculty of Alfred University

A Brief History of Mathematics in Ancient Greece and the "Eureka!" Moment for Modern Mathematics

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In Partial Fulfillment of
the Requirements for
The Alfred University Honors Program

29 April 2015

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ACKNOWLEDGEMENTS

I would like to thank everyone who has helped me through this process, especially my committee: Dr. Darwyn Cook, Dr. Joseph Petrillo, and Dr. Sandra Singer. I'd also like to thank the Alfred University Math Department for their support throughout my years at AU, Dr. Nathan Reff for introducing me to research in mathematics, Alyssa Waller for listening to me go on rants about ancient mathematicians, and my mom for all that she has done for me.

Finally, I would like to dedicate my thesis to my high school math teacher, John Baer. His encouragement in mathematics is what has inspired me to pursue my degree. I met him first when my older sister brought me to her class and had me recite all of the square roots she had taught me. Five years later, Mr. Baer was my teacher for 7th/8th grade math and again for AP Calculus. He has been a great role model, always supportive of my endeavors, and knew I wasn't being sarcastic when I said "Rolle's Theorem is cool."

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SUMMARY

The following paper offers an analysis of the history of mathematics in Ancient Greece. Its main focus is on five mathematicians from 600 BCE to 200 BCE: Thales, Pythagoras, Euclid, Apollonius, and Archimedes. After looking at the history of mathematics and the achievements of these mathematicians, I became aware of many factors that likely hindered these mathematicians from accomplishing even more. When I realized that many of these interferences were related to Ancient Greek culture, I was interested in examining the progress mathematics is making today. My thesis looks at the restrictions of the past and then looks at all of the opportunities for mathematics today.

INTRODUCTION

The field of mathematics is as ancient as it is broad. The human mind is naturally mathematical; numbers and logic rule our world. However, the mathematics we have today was not born overnight. It has been thought of and developed for centuries. While a variety of societies, both ancient and modern, have aided in the development of mathematics, the Ancient Greeks had a significant impact in the field. Those in Ancient Greece had a passion for knowledge and strived to learn all that they could about the world around them, and then apply it to general situations or situations that cannot be observed (Katz 33).

While the mathematicians of Ancient Greece contributed much to mathematics, mathematics of the time had the potential to go further than what the Greeks eventually accomplished. Several factors created a roadblock for those mathematicians that prevented them from developing mathematics into what we know it to be today. These factors come from a variety of categories, but include philosophy and some interesting demographics such as age.

Although we benefit from the accomplishments of ancient mathematicians, we can now see that these accomplishments were restricted by certain ideals of the Ancient Greek culture. Noticing the restrictions that the Ancient Greeks experienced in their attempts to advance mathematics, it is interesting to look at the progress made in modern mathematics.

The Ancient Greek world was the perfect place for new mathematical developments to take off. Greece itself is a large peninsula, therefore surrounded by the sea, which enabled the Greek world to spread all throughout and around the Mediterranean (Katz 33). Starting in the early 6th century BCE, scholars started to ask why things in the world were the way they were, and began to look for explanations that differed from mythological explanations (Calinger 61).

When these questions arose, the Greeks started to develop logical reasoning (Katz 43). This logical reasoning focused on geometry and was used to prove what was observed in the world (Katz 33). So, not only were the scholars of the time looking for explanations, they wanted to prove that each explanation was correct. Working in a society that encouraged discussion, it is expected that the Ancient Greeks would want to be able to prove their findings. Thus, the Greeks' admiration for debate led to the development of the mathematical proof.

MAJOR CONTRIBUTORS TO GREEK MATHEMATICS

Thales, Pythagoras, Euclid, Archimedes, and Apollonius are five of the most influential Greek mathematicians whose work encompasses some of the greatest achievements of mathematics from their time, and ours. Whether these contributions were in theoretical mathematics, geometry, or astronomy, they all made great strides in the field of mathematics. Some of these discoveries are still in use today as they were founded then, while others have either been refined or used as stepping-stones for later discoveries.

Starting around the sixth century BCE, attitudes toward learning began shifting in Ancient Greece. Right at the cusp of this shift lived Thales of Miletus (c. 625–547 BCE) (Calinger 64). Thales is credited with writing the earliest geometric proof (Katz 33). As Aristotle noted, "To Thales … the primary question was not *What do we know*, but *How do we know it*, what evidence can we adduce in support of an explanation offered" (qtd. in Dantzig 20). Mathematics is often thought of as a utilitarian field of study; Thales was one of the first people who began to look at how and why mathematics works instead of looking at its applications. There is not much known about Thales' life since much is disputed among historians. Some claim he was a patriarch of a large family, and others say he never had children (Dantzig 22). What is certain is that he began his mathematical career in Miletus and then traveled to Egypt and Mesopotamia (Calinger 65). Even though we do not know details about his private life, there are a few things known about Thales, and the impact of his accomplishments still resonates today.

Thales made an assortment of mathematical discoveries. While some of Thales' discoveries greatly changed the view of mathematics, others were more practical. During his time in Egypt, Thales was given the task of determining the height of the Great Pyramid. He was able to do this even though so many priests before him were not (Dantzig 46). Thales' approach to this challenge was to wait until the length of a shadow of a person was equal to his or her height (Dantzig 47). Using this method, the height of other objects could be determined without having to climb to dangerous heights.

Thales is known as one of the Seven Sages, or Seven Wise Men, of Greece (Dantzig 21). While he is known for his assistance in the development of the proof, Thales also provided the following five geometric propositions that are still in use today: A circle is bisected by its diameter; base angles of an isosceles triangle are equal; if two lines intersect, then the vertical angles formed are equal; angle-side-angle being able to prove two triangles congruent; and every triangle inscribed in a semicircle is a right triangle (Calinger 66). It is not certain how Thales proved these propositions because much of his work has been lost, but they have promoted the evolution of logical mathematics.

The Pythagoreans are one of the most fascinating groups of Ancient Greek mathematicians. They were a group of disciples of Pythagoras (c. 572-497 BCE), but the group continued on long after his death (Katz, 36). The teachings of Pythagoras and his followers were not only about pure mathematics and reason; they also integrated mysticism with philosophical speculation to create a world of numbers that explained the universe (Callinger 68). For being a group of mathematical intellectuals, the Pythagoreans had some characteristics of a religion. Pythagoreans were vegetarians, did not drink wine, and believed in reincarnation along with other cult-like behaviors (Calinger 69).

The Pythagoreans also had numbers that were sacred to them, and each number had a meaning. For example, the integer 10 is considered the most perfect number to the Pythagoreans because it is the sum of the first four integers, base 10 is used in the decimal system, and a tetractys (Figure 1) is formed using ten points (Calinger 69). Other numbers considered to be perfect by the Pythagoreans are integers that are equal to the sum of their divisors, such as 6 (1+2+3) and 28 (1+2+4+7+14) (Calinger 72). The Pythagoreans' philosophy both helped and limited their mathematical progress.

The Pythagoreans are interesting not only for their unusual lifestyle and their combination of the mathematical with the metaphysical, but also for having a theorem attributed to them for something that they more than likely did not discover (Calinger 68, 72). Almost everyone who has taken an algebra or geometry class has heard of the Pythagorean Theorem, which would make Pythagoras one of the more familiar mathematicians to people today. Interestingly enough, it is doubtful Pythagoras and his followers are actually responsible for the Pythagorean Theorem (Calinger 76). Pythagorean triples (a, b, and c where $a^2 + b^2 = c^2$) were known since around the eighteenth century BCE in Mesopotamia (76). While we are unable to attribute with certainty the concept of the Pythagorean Theorem to the Pythagoreans, they were the first to actually prove it instead of solely utilizing it. From a mathematical perspective, the proof is the part that is most significant, and a claim, even if it is true, cannot actually be made until it has been proven. So, while the Pythagoreans were not the first to encounter Pythagorean Triples, they were the first to prove the Pythagorean Theorem; thus it is credited to them.

The most significant piece of Pythagorean arithmetic is even and odd theory. The development of an even and odd theory became the foundation for many mathematical advancements, especially for the discovery of incommensurable magnitudes and numbers (Calinger 74). Incommensurable numbers are numbers that cannot be expressed as a ratio of whole numbers, and incommensurable magnitudes are magnitudes that have no common measurement. The Pythagorean even and odd theory is composed of the following three theorems regarding even and odd integers: (1) the sum of two even numbers is also an even number, (2) the product of two odd numbers is an odd number, and (3) when an odd number divides an even number, the odd number will also divide the half of the original even number (74).

Distinguishing evens and odds and creating these theorems enabled larger discoveries to be made. The Pythagoreans used even and odd theory to prove that $\sqrt{2}$ is incommensurable with 1, or that $\sqrt{2}$ and 1 have no common measurement, which can be seen in the relationship between the side and diagonal of a square (Figure 2). Showing that $\sqrt{2}$ is incommensurable with 1 also proves that $\sqrt{2}$ is irrational because it cannot be written as a ratio of integers.

The proof that $\sqrt{2}$ is irrational is one of the most important proofs of the time, but it did go against the Pythagoreans' strong feelings that numbers, specifically positive integers, underlay and create the geometry that rules the universe (Calinger 71). The Pythagoreans, specifically Hippasus of Metapontum (c. 5^{th} century BCE), are credited for the discovery of incommensurable numbers (Calinger 74). The Pythagoreans and other mathematicians of the time attempted to find some measure for a square where the sides and diagonal can be divided into equal units of the same size, also referred to as countable or commensurable (Katz 39). Unfortunately for them, this is impossible to accomplish. The side and diagonal of a square are incommensurable, or having no standard measure in common (39). This comes from the fact that the unit 1 is incommensurable with $\sqrt{2}$, or equivalently $\sqrt{2}$ is irrational (39). Hippasus used theorems of odds and evens in his proof by contradiction, or reductio ad absurdum (Calinger 74-75). Using this method, Hippasus encountered a contradiction regarding even and odd theories the Pythagoreans had already proved. The claim that $\sqrt{2}$ is commensurable with 1 led to this contradiction and therefore they knew it must not be true.

Most people have heard of the Pythagoreans, but the mathematician with arguably the most dominant presence in mathematics today is Euclid. Euclid was born around the mid-4th century BCE and died somewhere in the mid-3rd century BCE, and he was most likely one of the very first mathematicians to study at the Library at Alexandria (Katz 51). His most famous work, *The Elements*, is filled with definitions, axioms, theorems, and proofs, many of which are still in use today (51). A textbook that has possibly been in use for the longest amount of time globally, *The Elements* was purposed to lay out what people learning mathematics should study. While *The Elements* simultaneously advances mathematics and becomes a foundation of learning for years to come, it primarily focuses on geometry. Focusing almost entirely on geometry made it difficult for anyone to get away from and move on to other disciplines of mathematics because it was widely used and worked well as a textbook. However, the accomplishments contained in *The Elements* are a great example of mathematical accomplishments of Ancient Greece.

The Elements contains fragments of Euclid's original works that have been compiled into thirteen books. Each book concentrates on a different subject (Katz 52). The books of *The Elements* are broken up into different topics as follows: Books I through VI focus on 2-

dimensional geometric magnitudes; Books VII through IX deal with the theory of numbers; Book X discusses commensurability and incommensurability; Book XI is about 3-dimensional geometry; Book XII contains the method of exhaustion for 2 and 3-dimensional objects; and, lastly, in Book XIII Euclid shows the construction of five regular polyhedra (Katz 52). Several of the books begin with definitions and axioms that are used throughout the book (53). Each result in *The Elements* comes from a previously proven result similar to a domino effect (53). While all of the books in *The Elements* form a foundation for geometry, several of the books have greater implications.

Book X is often considered the most important by historians (Katz 81). It is the longest and most organized of the books in *The Elements* and deals with a previously controversial subject: incommensurable magnitudes (Katz 81; Dantzig 103). Proposition X-9 is a generalization of the discovery of incommensurability of the diagonal of a square with its side, or the Pythagorean proof showing $\sqrt{2}$ is irrational (Katz 82). Euclid credits the generalization of the proof to Thaeatetus who showed incommensurability of the square root of non-square integers with 1 (Katz 82). Equivalently, these proofs show that the square roots of non-square integers are irrational, a great triumph in early mathematics.

Euclid's Book XII uses the method of exhaustion to estimate formulas for areas and volumes of different shapes. This method involves filling an irregular figure with smaller figures of known area or volume in order to estimate the area or volume of the irregular figure, respectively (Shankar 1). The method of exhaustion was first employed by Eudoxus (408-355 BCE), who is known for combining mathematics and astronomy (Katz 139). It is the very same method later used by Archimedes, but on a slightly different level. While Euclid approximated volumes and areas using a finite number of shapes, Archimedes used the concept of infinity.

Infinity greatly terrified mathematicians of the time due to Zeno and the paradoxes he presented involving infinity (Katz 45). Though, Euclid used the method of exhaustion up until a potential infinity—knowing that he could add an infinite amount of shapes, but would stop at a certain point—his use of Eudoxus' method helped to popularize it as *The Elements* became a staple text in mathematics.

Book XIII is the final book of *The Elements*, and focuses on the construction of five regular polyhedra: the cube, tetrahedron (also known as a triangular pyramid), octahedron, dodecahedron, and the icosahedron, (Figure 3) (Katz 87). All of the other books in *The Elements* lead up to these polyhedra and how they relate to spheres. By inscribing each polyhedron into a sphere, certain comparisons were made between them and the sphere they were inscribed within, such as the length of the edge of a polyhedron to the diameter of the sphere (87). Ending *The Elements*, Euclid proved that there are no other regular polyhedra than the five he constructed (88). Euclid contributed much to mathematics, but by far his greatest contribution was *Elements*.

Apollonius (250–175 BCE) was first recognized for his work with astronomy, and then later became known for his work with conic sections (Katz 114). Similar to Euclid, Apollonius is known mostly for one work. Apollonius' major work, *Conics*, was written when he was quite old (Netz 198). Apollonius used what we now call a double oblique cone—two cones atop each other point-to-point (Figure 4)—with different planes slicing through the solid to generate four curves: the parabola, the circle, the ellipse, and the hyperbola (Katz 114). Even though it is uncertain how exactly Apollonius was able to show and prove his findings (Katz 114), they are still applicable and contributed much to geometry. Similar to Euclid's *The Elements*, *Conics* is also broken up into several different books. Books I through IV revisit the fundamental theorems of conics and present them in a way similar to Euclid's first few books (Calinger 177).

Books V through VII are more specialized to the work of Apollonius (Calinger 178). The specialized topics covered in these books are maxima and minima, equal and similar conic sections, determinations of conjugate diameters of a central conic, and determinate conic problems (Calinger 178).

In *Conics*, Apollonius aimed to generalize theorems for circles and apply them to other curves (Katz 120). Before Apollonius universalized these theorems to other curves, he first demonstrated the construction of each of the four curves in conics. One of these curves is the hyperbola which is discussed in Book II. Hyperbolas are different from other conics because they are composed of two parts and are bounded by asymptotes. Asymptotes are lines that a function approaches, but never reaches (Figure 5). In Proposition II-1, Apollonius constructs the hyperbola and asymptotes, and later in Proposition II-4 he demonstrates how to construct a hyperbola given only a singular point on it and its asymptotes (Katz 119). Later in *Conics*, Apollonius used the curves he created in solving various geometric problems that were previously unsolvable (Katz 125).

Archimedes (c. 287–212 BCE) is considered to be one of the most influential mathematicians of all time (Katz 95). He was a Renaissance man centuries before the Renaissance, for not only was he a mathematician, but he was also an inventor of many devices (95). His discoveries were giant leaps in multiple fields, and his findings in mathematics are some of the greatest. Archimedes differed from Euclid and other mathematicians in that he would present how he came to making a discovery and his method before he would prove it (103).

Archimedes is most commonly known today for his work for Hiero, King of Syracuse. Archimedes spent most of his life solving practical problems for Hiero, including creating war machines and determining the percentage of gold in Hiero's crown (Katz 94, 97). As the story famously goes, Archimedes was in the bath when he realized he could use water displacement to help determine the amount of gold in Hiero's crown. He then became so excited that he jumped out of the bath and ran around town naked yelling "Eureka!" meaning, "I've found it!" (Katz 94). While most of Archimedes' career was focused on work for Hiero, Archimedes also served the mathematical world as a whole.

Archimedes commonly used Eudoxus' method of exhaustion, that is, filling an irregular shaped area with smaller figures with a known area to estimate the area of the irregular shaped figure (Shankar 1). This method was also commonly used by Euclid (Shankar 1). With this method, Archimedes utilized curved shapes to approximate areas and volumes, and derived a formula to find the area of a section of a parabola. He proved that the area under a section of a parabola is $\frac{4}{3}$ multiplied by the area of the triangle circumscribed in the shape (Figure 6) (Rehmyer). In Archimedes' *Quadrature of the Parabola* and *The Method*, he extended the method of exhaustion to find the volume of a sphere and a paraboloid (Shankar1).

One of Archimedes' greatest advancements appears in his work, *The Method*. What makes his work more interesting is that it was lost for many years. While it was previously thought that none of the ancients were close to developing calculus, Archimedes was actually very close. We only have *The Method* today due to a series of peculiar events (Rehmyer). It was used 700 years ago by a monk as scrap paper for a prayer book, which was found in a library in Constantinople at the turn of the 20^{th} century, and then later kept by a French family who thought it was just a prayer book until they took it to an antique show (Rehmyer). *The Method* was discovered in 1899 but then lost (Katz 103). It was finally rediscovered in 1998 (Katz 103). The method of exhaustion was used throughout *The Method*.

Generally, exhaustion would be used until a certain approximation was reached, however Archimedes used this method an infinite number of times, mimicking integration (Rehmyer). For example, when taking the volume of a figure shaped like a fingernail, he looked at a 2-dimensional slice through the solid instead of using the usual method of exhaustion. He also created the same slice in the irregular-shaped solid, but closed it off with straight edges, which created a triangular figure (Figure 7). Archimedes then compared the slice in the original figure and the closed off figure (Rehmyer). Archimedes argued that the original shape and the triangular prism he created around it had the same number of slices—an infinite amount—and used this comparison to calculate the volume of the curved shape (Rehmyer).

The most remarkable part about Archimedes' findings is that it shows his understanding of infinity (Katz 107). While Aristotle distinguished between two types of infinity, actual and potential infinity, at the time only potential infinity was understood and it was used almost to the exclusion of actual infinity. Potential infinity is used in the method of exhaustion because it claims the process can go out to infinity, but at a certain point stops and says that the approximation is close enough (Rehmyer). In *The Method* Archimedes said that the infinite sets are "equal in multitude" (Katz 107). Archimedes demonstrated a deeper understanding of infinity than many others from his time because he was able to use the idea of actual infinity in *The Method* as opposed to being satisfied with a less accurate answer. Since Archimedes stated that particular infinite sets were equal, perhaps he had even considered different infinite sets that were not equal (Katz 107). While his proof of using exhaustion to infinity is not completely valid, Archimedes believed his method should work. His method did work, but it was not fully proven until Newton and Leibniz and the development of calculus (Rehmyer). What became a roadblock for other scholars at the time did not hinder Archimedes. The unlikely story of how

The Method was lost and found today raises the question: What other previous discoveries have been made and then unfortunately lost?

HINDRANCES TO THE DEVELOPMENT OF MATHEMATICS IN ANCIENT GREECE

While many great mathematical contributions were made between the 6th and 2nd century BCE (such as new methods, theories, and discoveries) it is important to uncover factors that prevented the Greeks from advancing mathematics even further. For example, Archimedes came close to approximating pi; he calculated it to be between $3\frac{10}{71}$ and $3\frac{1}{7}$. But the lack of regular use of algebra and numbers did not allow him to calculate a more precise value (Shankar 1). The Greeks had no notion of algebra until sometime around 300 BCE (Katz 66). Before this time, they only used geometric terms for working expressions that represented magnitudes (Katz 66). Additional factors that impeded the growth of mathematics were mostly cultural. Some of these hindrances affected Greek mathematics as a whole, while others uniquely impaired specific individuals and/or groups.

First, there are a few factors to consider that affected Greek mathematics in totality, and not particular mathematicians. The most obvious reason for the underdevelopment of Greek mathematics is simply that there were not many mathematicians around. According to *Science and Mathematics in Ancient Greek Culture*, it is thought that the number of mathematicians throughout classical antiquity was about one thousand (Netz 202). For a period that spans over 1000 years, that is very few mathematicians. Netz estimates that there were potentially three mathematicians born per year, which implies that there were maybe 100 active mathematicians alive at one time (203). The Greek-speaking world being as vast as it was also indicates that these potential 100 mathematicians might have been considerable distances away from each other. Unlike today, there was no effortless way for people to communicate over great distances, so it would have been extremely difficult for these mathematicians to find or converse with one another. Furthermore, the death of just one mathematician had the potential to seriously affect the link between all of them (Netz 203). For example, Conon of Samos (c.280 BCE – c. 220 BCE) was one of Archimedes' main mathematical correspondents. After Conon of Samos died, Archimedes was desperate to find someone with whom he could communicate his results (Netz 203). In an effort to maintain some correspondence, Archimedes first contacted Dositheus of Pelusium, a former student of Conon (Netz 203).

In Archimedes' *On Spirals*, he writes to Dositheus, "though many years have elapsed since Conon's death, I do not find that any one of the problems has been stirred by a single person" (Archimedes 80). Again, Archimedes had almost no one to talk to about his discoveries. Even after years of his problems being exposed to the public, no one had even tried to answer them. Not only could the death of one person affect the existing, fragile network of mathematicians, but if a city had only one mathematician, his work would most likely die with him (Netz 215). The papyrus used at this time was not sturdy enough to last more than a generation under usual conditions, so work had to be rewritten in order to survive (Netz 215). Without someone to finish carrying out the work of another, or to reproduce the work once the papyrus became too timeworn, all of the findings made by a person could vanish until someone else made the same discovery all over again. The scarcity of mathematicians is arguably one of the major reasons why mathematics did not progress any further than it did in the Ancient Greek world.

Second, in order to even be a mathematician during that time, there were a few qualifications one had to meet. To even be exposed to mathematics, a person most likely had to be male and rich with the most exceptional work occurring at an older age (Netz 201). Without meeting these criteria, a person of antiquity had a very slim chance of getting into the field of mathematics. Not all mathematicians of the time were men. There are a few women who studied mathematics in Ancient Greece, such as Hypatia from the 5th century CE, and Pandrosion about a century earlier. Still, women made up a very small fraction of the group (Netz 197). Further, although there were some younger Ancient Greek mathematicians, such as Theaetetus and Pythocles, many more were older when they produced some of their greater work (Netz 198-199).There were so few mathematicians because it was such an inaccessible discipline. There were no careers in mathematics at the time, and there were no mathematical faculties (Netz 208). Only those who could afford to dedicate themselves to the subject were able to be involved with it. A person then had to have money and come from a substantial family.

While some factors that hindered ~~of~~ mathematical development were widespread in the culture, others were more specific to individuals. Largely, these hindrances were metaphysical beliefs. The Pythagoreans exemplify many of the philosophical and mathematical roadblocks that many of their contemporaries encountered.

The Pythagoreans were very focused on the universal implications of mathematics. They believed that geometry and numbers had to be understood for the universe to be interpreted (Calinger 70). However, they were so devoted to natural numbers that they denied anything that went against their beliefs. Their faith in geometry and whole numbers was so extreme that there is a legend about the reaction of the Pythagoreans when Hippasus proved $\sqrt{2}$ is incommensurable with 1. When Hippasus claimed $\sqrt{2}$ to be incommensurable with 1, they attempted to throw him overboard and drown him for making a claim that undermined their belief that everything is a number or ratio of whole numbers (74). While this story is a bit extreme, it does portray the ferocity of some of the philosophical beliefs at the time, and how they interfered with mathematical discoveries. If Hippasus had not been able to accept that what he and the Pythagoreans speculated could be false, then proving $\sqrt{2}$ to be irrational, and later the proof of other non-square integers as irrational, would have been delayed.

Along with having too much faith in geometry to solve anything mathematical, there was a preoccupation with the circle in geometry. This can easily be seen in the work of Eudoxus and Apollonius. In his *Commentary* on Aristotle's *On the Heavens*, Simplicius states, "Plato … set the mathematicians the following problem: What circular motions, uniform and perfectly regular, are to be admitted as hypotheses so that it might be possible to save the appearances presented by the planets?" (qtd. in Katz 133). Plato initiated the movement to try explaining heavenly bodies using regular shapes and circular motions. Those that attempted to solve Plato's problem were very limited because they considered the universe to be perfect and never changing, and what they thought to be perfect was to have spherical objects in the universe move in circular movements (Katz 137). The Ancient Greek aesthetic of perfect circles and spheres played a key role in the creation of models of the universe that were incorrect. Astronomers and mathematicians went to great lengths in order to explain the movements of heavenly bodies while retaining their belief in a perfect universe with circular movements.

To compensate for movements of the sun, stars, and planets that did not seem to rotate around the earth in a perfect circle with constant velocity, astronomers and mathematicians used multiple spheres in a variety of configurations (Figure 8) (Katz 139-141). It began with Eudoxus using multiple spheres placed inside each other with separate axes of rotation to explain the movements of the sun and the moon. In Eudoxus' model the sun required only two spheres to explain its movements, while the moon required three (Katz 139). These spheres functioned similar to a gyroscope where the earth is the unchanging middle. In his model of the sun, the sun rotates around the equator of the inner sphere that is inclined at an angle within the outer sphere that is also rotating (Katz 139). The model of the moon was similar, but required another sphere (Katz 140). While it is most likely Eudoxus did not believe these spheres had actual existence as much as he just used them for computational purposes, they became incorporated into astronomical thought through the sixteenth century (Katz 140).

Eudoxus' spheres did not fully explain universal phenomena, and approximately 150 years later Apollonius also attempted to create a model that solved the problem proposed by Plato (Katz 140). Apollonius created two models that attempted to solve Plato's problem. These models are the eccenter model and the epicycle model. For the eccenter model, Apollonius merely shifted where the center of the sun's orbital path was with respect to the earth so that the earth was no longer in the center of the orbit (Figure 9) (Katz 141). This allowed the sun to move around uniformly in a circular motion, but could explain some of the differences in days for each of the four seasons (141). The epicycle model is also used to explain disparities between the model of the universe and the actual universe. Instead of moving the center of the circular orbit, this model adds a smaller orbit that follows the larger orbit in one direction, while the smaller orbit moves in the opposite direction (Figure 10) (140-141). While both models make an effort to explain the intricacies of the universe, a more complicated model can be created by combining the two (Figure 11).

The Ancients went to great lengths to preserve their original assumptions that everything revolves around the earth and goes in perfect circles. Knowing today that that is not the case, it all seems rather silly. Aristarchus of Samos (c. 310–230 BCE) believed that the earth revolves around the unmoving sun (Katz 136). His idea seemed so ridiculous to other astronomers at the time that they called him impious (136). It is extraordinary that there was such a dependence on and dedication to having everything worked out with perfect circles. Thinking maybe everything orbited in another shape, such as an ellipse, was almost unheard of. The model of the universe created during this time would rather force circles to work for everything rather than consider, perhaps, that a circle is not the right fit. The Ancient Greeks were especially focused on geometry along with finding perfect numbers and shapes, as well as having the universe be explained by these things. However, this focus blinded them to the very real possibilities that not everything is perfect, or at least in the way that they wanted.

THE PROGRESS OF MATHEMATICS TODAY

Mathematics today is an ever-growing field. Onlookers of the mathematical world may not realize it, but there is constant research in all disciplines of mathematics. There are thousands of new PhD's in mathematics each year, with each person writing a dissertation on new mathematical concepts. After seeing what prevented the progression of mathematics in Ancient Greece, it is important to consider why mathematics is advancing so quickly today.

Looking at how research is conducted is important in order to know how we develop mathematics today. While there are many ways to approach mathematical research, it all starts with learning how to get in the right state of mind which begins with learning how to prove things that have already been proven. Often, a person pursuing higher levels of mathematical education, such as graduate school, will be given problems to work on either to prove a statement or to come up with an example of a particular occurrence in mathematics (Lady). These kinds of questions get people to think more theoretically so that they may be able to create their own problems or situations that do not have any solution yet. In the article "How to Succeed in Mathematical Research," there are many tips for someone who is conducting mathematical research. The article encourages those participating in research to be determined, but also flexible when researching (Etingof). Being stubborn forces a person to keep trying when facing adversity in his or her research, but being flexible allows a person to take different approaches or recognize when he or she is stuck. Another point this article makes is for the researcher to ask for help, advice, or feedback from other people (Etingof). Mathematical research is more social than other types of research, because it involves discussion. It is difficult to come up with an exact method for performing mathematics research. However, practice, being determined but flexible, and discussing ideas and findings are key aspects of conducting successful research in this field.

Today, there are many people involved in mathematics. As of 2011 it was estimated that just in the United States there were approximately 202,667 people active in a mathematical occupation; this makes up 2.8% of the science, technology, engineering and mathematics (STEM) workforce (Landivar, *Relationship* 5). Approximately 2,450 of those people referred to themselves as mathematicians (Landivar, *Relationship* 5). From those numbers, there seems to be no lack of mathematicians today. The field of mathematics is saturated with mathematicians who may help educate, share findings, and just communicate with each other. Indeed, not only are there many people working in a mathematical profession, there are also many people with a mathematical degree. In 2011, of people aged 25 to 64 working in a STEM field, 4.6% of them had at least a bachelor's degree in Computer Science, Mathematics, or Statistics (Landivar, *Relationship* 14). From this data, it is clear that not everyone who studied mathematics became a mathematician, but there are people out there with the education and ability to pursue the field.

Women have a significant presence in the field of mathematics today. According to a national study published in 2011, 47% of mathematical workers in the United States are women (Landivar, *Disparities* 6). This implies there were approximately 95,253 female mathematicians in the United States in 2011. There are so many female mathematicians in the U.S. today alone, but considering the world in its entirety there is potential for many more. Thus, at least in the U.S., many people are participating in mathematics regardless of gender.

Today, one might think that social standing and wealth might still be obstructing the progress of mathematics. Although there are more equitable circumstances today for those of a low socioeconomic status, it is still very possible that not all of those who could flourish in the field are presented with opportunities to study mathematics. It is also difficult to evaluate the economic status of today's documented mathematicians leading up to their involvement in the field. Also, for each mathematical occupation, there is a vast range of salaries. While there may be some variety due to location, the largest changes in salary come from years of experience.

Looking at the socioeconomic status of students who attend a university shows that the opportunity exists for those of a low-income and low-status to study mathematics at a higher level. Overall, there has been a steady increase of low-income students enrolling in a two- or four- year college over the past 40 years (Desilver). But only 50.9% of low-income households were enrolled in a two- or four- year college in 2012, compared to 64.7% of middle-income households and 80.7% of high-income households, where the bottom 20% of family incomes represents "low-income" and the top 20% represents "high-income" (Desilver). While there is a distinct gap between high- and low-income enrollments, low-income enrollments still exist and are on the rise. Whereas the opportunity for a higher education may not be as promising for low-income students today, it is still very possible to achieve one. But there is definitely room for improvement to include more of those who may not have the funds to attend college, but there are more opportunities becoming available for those of lower incomes to become a mathematician.

Today, people of all ages have the chance to be involved in mathematics. There is almost no limit on age for anyone to study mathematics. Children are introduced to basic arithmetic at a young age and continue with mathematics at least through most of high school. The opportunity also exists for people of an older age to go back to school, especially colleges and universities. Undergraduate and graduate students assist their professors in research, and many summer research opportunities are also available. With schools being open to both the young and the old, there is not truly an age barrier preventing someone from studying math. However, one could argue that today it is more beneficial to be young and studying mathematics than it is to be older. Many people say that a person's best work occurs in their early twenties, and in order to win the Fields Medal (a lucrative prize awarded to mathematicians for their work) a person must be under the age of 40. While there is no social stigma and there is an opportunity for people of any age to participate in mathematics, conditions today may favor the young over the experienced.

If religious beliefs were a barrier to mathematical development in the ancient world, today's vast spectrum of beliefs allows for challenges to even dominant perspectives. For example, when one person is prevented ~~by~~ from solving a problem because of his or her personal philosophies, another person may have a new perspective. Today, there are various cultures and religions, more or less all of which study mathematics. In 2008, an estimated 228,182,000 adults in the United States identified as one of over 30 denominations of Christianity, according to the

U.S. Bureau of Labor Statistics. From the same data, it is also estimated that there are 8,796,000 adults who identify with Judaism, Hinduism, Islam, Wiccan, Pagan, or other religions (U.S. Bureau of Labor Statistics). Further, in this data an estimated 34,169,000 adults identify with no religion (U.S. Bureau of Labor Statistics). With such a diverse group in the U.S. alone, it is very likely that there are differing philosophical beliefs among them. This suggests that what may be a philosophical barrier for one person, may not be there for another.

CONCLUSION

Overall, it is very difficult to step back and evaluate modern mathematics to find what may or may not be stopping its evolution. We can look back at what has stopped others in the past and also look at how far mathematics has progressed today. So even though we see what prevented the Ancients from making discoveries such as calculus, we are able to see how far mathematics has progressed and what is allowing it to advance further.

The mathematicians of the Ancient Greek world brought about a new perspective and new mathematical developments. While their findings were very significant, ancient mathematicians might have made even more progress had there not been so few of them and had they not been so focused on geometry and perfect shapes, numbers, and ratios. The focus on geometry is natural, for it is easy to observe and useful for measurements and engineering new devices, however geometry ignored much of theoretical mathematics. Today, we might be blind to see that there are other ways to use and apply mathematics, just as the Greeks were blinded by their beliefs. Mathematics in Ancient Greece focused almost solely geometry, but today mathematics attempts to bring as many topics together as possible. While there were many hindrances along the way, mathematics has made great strides and progress to get where it is today and the environment modern mathematicians have been cultivating will allow the field to continue to grow.

APPENDIX

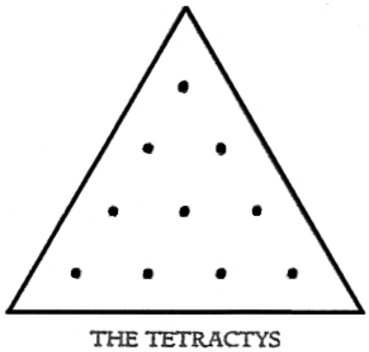

Figure 1

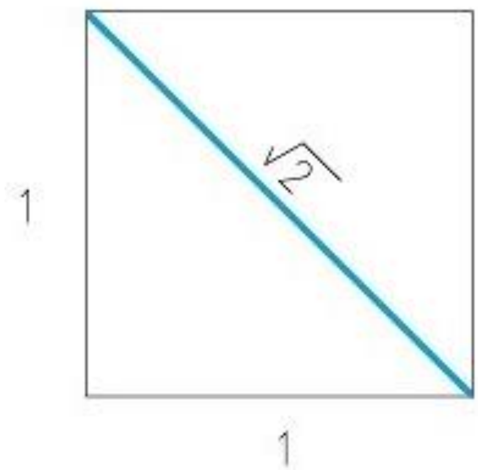

Figure 2

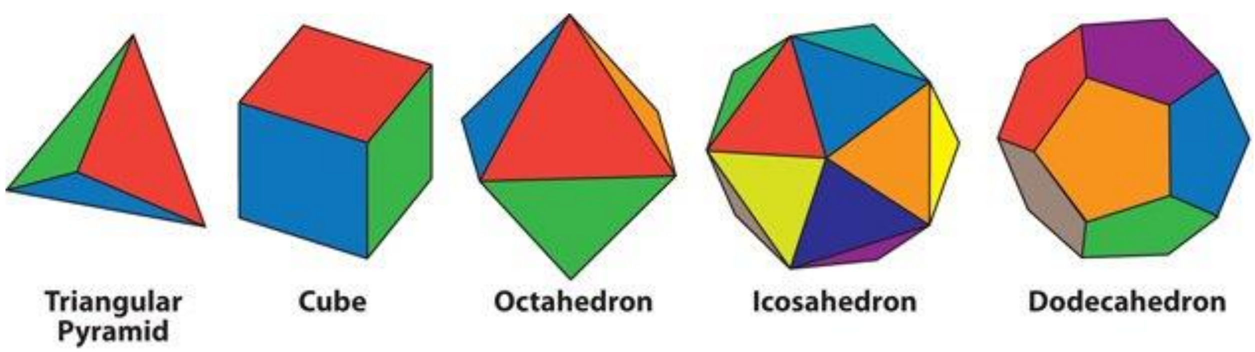

Figure 3

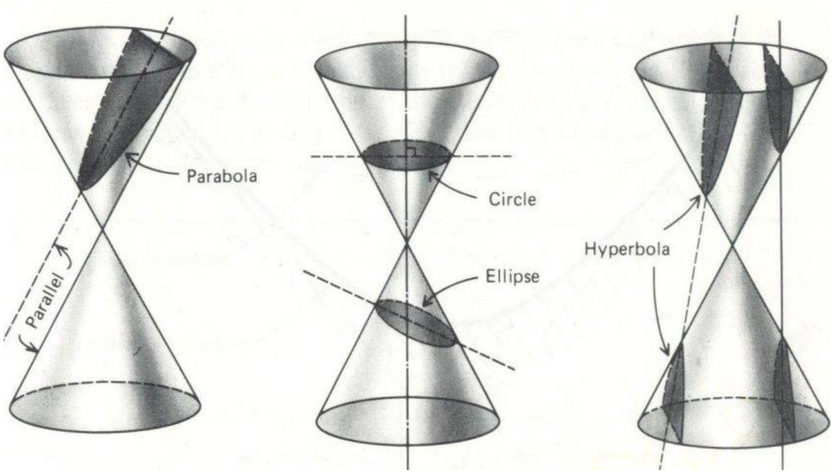

Figure 4

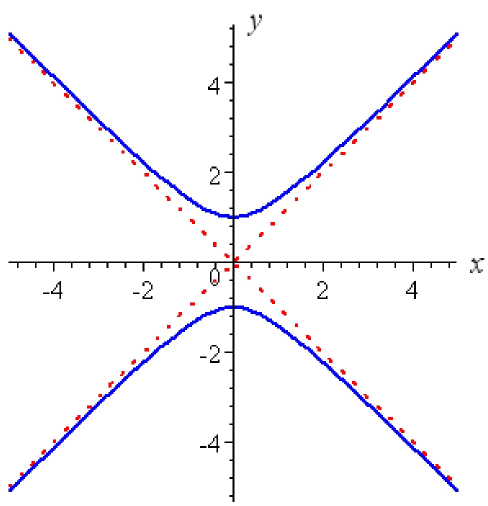

Figure 5

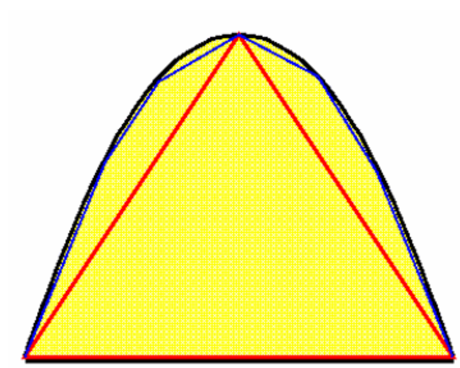

Figure 6

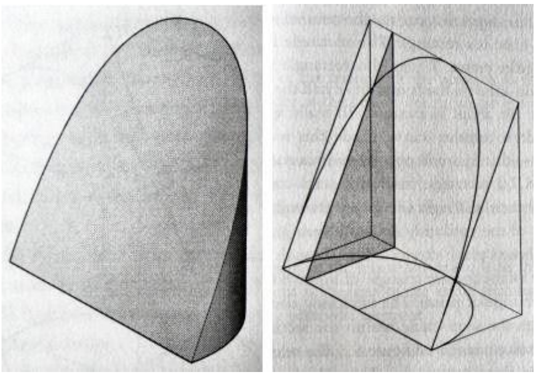

Figure 7

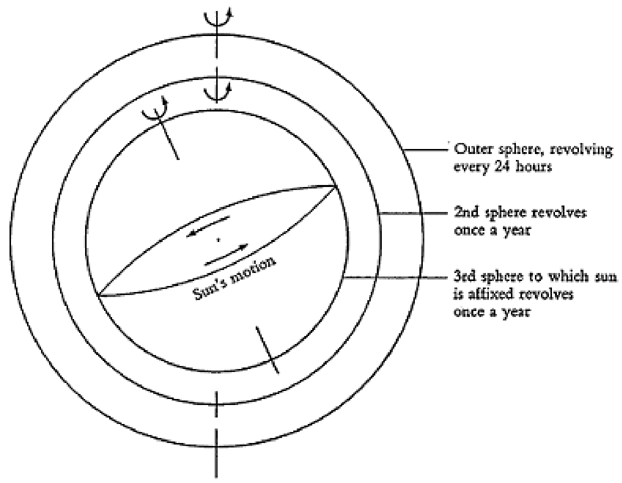

Figure 8

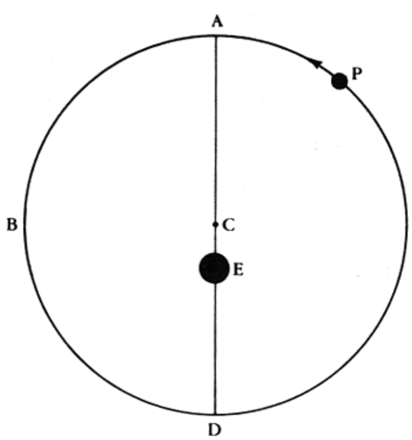

Figure 9

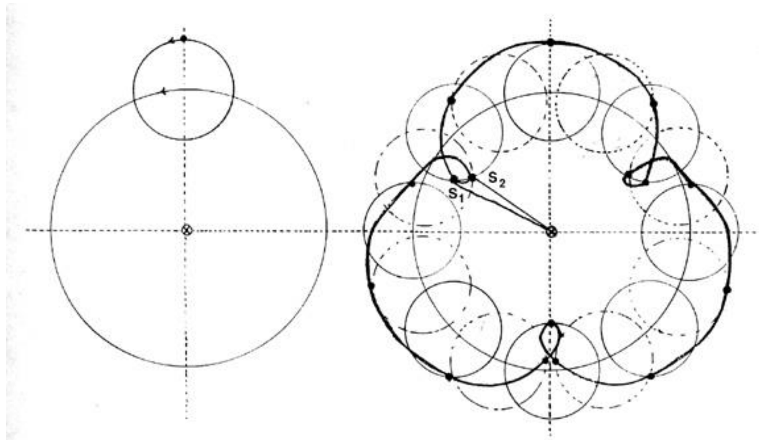

Figure 10

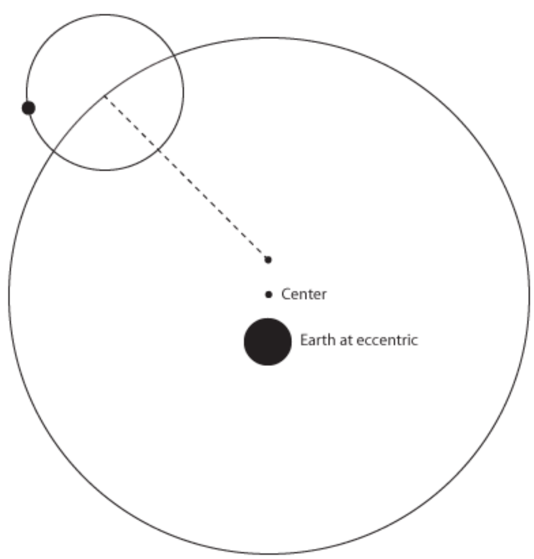

Figure 11

NOTES

1. Figures 1 through 5 are from the following sources, respectively:

"Manuscript of Sacred Geometry." *Metempyrion Foundation*, n.d.. Web. 27 April 2015.

"What Does the Square Root of 2 Look Like?" *Squarerootcalculator.co*, n.d.. Web 27 April 2015.

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2. Figures 6 and 7 are from Rehmyer's "A Prayer for Archimedes."

3. Figures 8 through 11 are from the following sources, respectively:

"Astronomy." *E-ducation.datapeak.net,* n.d.. Web. 27 April 2015.

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4. References used in writing this paper are the sources cited below, as well as the committee members.

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