A Thesis Presented to

The Faculty of Alfred University

DEVELOPMENT OF A TWO-PARAMETER MODEL TO DESCRIBE PARTICLE SIZE DISTRIBUTIONS

By

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ABSTRACT

Particle size distributions present a unique challenge for analysis and presentation and simply reporting the D_{50} value fails to capture any information that describes the width of the distribution. By fitting the particle size distribution to a statistical model, it is possible to describe a distribution with a two-parameter model, similar to that obtained from a Weibull analysis of mechanical testing data. In fact, as is demonstrated in this thesis, many native particle distributions actually fit a Weibull distribution, but when the distribution is scalped, as is common for industrial powders, the distribution is better described with a log-normal model. Both of these distribution types can be described by a mean and a modulus, and thus a two-parameter model. The two-parameter model can then be plotted on x-y coordinates to allow the tracking of particle size distributions for milling studies or for quality control purposes.

I. INTRODUCTION

Particle size distribution data are commonly collected and reported for many ceramic powders. This data is however underutilized and often does not allow for the comparison between various samples and powders in varying applications. Most commonly the particle size data is recorded through the value of the D_{50} or the mean particle size, but this only covers one aspect of an overall particle size distribution, effectively making it a poor marker for comparison. The hypothesis of this study is that regardless of the nature of the particle size distribution, be it scalped or native, it is possible to describe a particle size distribution with a two-parameter model through the use of statistical analysis.

I first became interested in this topic my sophomore year of the ceramic engineering program. During a lecture Dr. Carty described log-normal distributions and how they can be used to describe particle size distributions. He mentioned that this method should show the distribution as a line. I engaged in a conversation with him about describing each material by a slope of that line. Later that day, I received an offer to pursue research under Dr. Carty in a work study position later that day. This would be the spark that ignites not only my thesis research but also my love for research.

I began to do statistical analysis on various particle size distributions provided to me by Dr. Carty. The first problem to overcome was an issue in creating a probability axis in order to make a linear approximation. The way textbooks describe creating a log-normal distribution does not lend itself to use in Excel otherwise it is simply an axis generated that cannot create a slope as the space between numbers on the axis are not consistent. This issue was solved by applying a statistical analysis function from Excel that is used to compare many points of data by its percentage compared to 50%.

Once the ability to describe the percentage of mass below a particle diameter was developed it was applied to as many datasets as I could get my hands on. In this study, 86 historical particle size distribution datasets were obtained for various ceramic powders. These datasets were recorded using an x-ray sedimentation instrument. The collected datasets reported a D_{50} for the powders analyzed as well as reporting a cumulative mass finer than percent (CMFT%) related to a particle diameter measured.

It was discovered in this study that 64 particle size distributions fit a log normal distribution while 22 fit a Weibull distribution based on the R^2 value for the linear approximation. The distributions were affixed to a plot as a point described by their D_{50} and slope from a linear approximation of a log-normal distribution in order to analyze and compare various particle size distributions at once.

This procedure can be applied to various aspects of particle size studies. Industrial applications of this procedure could be used for quality control applications to facilitate tracking of particle size distributions of incoming raw materials. This can also be used to analyze and track particle size distribution changes during a milling study.

Particle size distributions are very easy to obtain. Many distributors of raw material will provide their clients with particle size distribution data. But if the distributor were to not provide the distribution data it is very easy to gather the data with the right equipment. The person looking to find particle size distributions doesn't require access to an x-ray sedimentation instrument as long as the powder size is larger than the colloidal limit. The simplest way to gather particle size distribution is through the use of sieves. Figure 1 shows a photo of an ASTM standard test sieve.

Figure 1. Example of a standard test sieve for particle size analysis.

Sieves are placed in a stack based on decreasing the size of the mesh as it falls. This stack is then placed on a vibratory table and after the stack has been agitated the mass of powder remaining in the mesh is measured in order to determine the mass of the powder that is within a specific size range. The ASTM sieve designations are shown in Table I.

Table I. ASTM Standard Sieve Designations

Sieve Designation	Opening Size
No. 4	4.75 mm
No. 5	4.00 mm
No. 6	3.35 mm
No. 8	2.36 mm
No. 10	2.00 mm
No. 12	1.70 mm
No. 14	1.40 mm
No. 16	1.18 mm
No. 18	1.00 mm
No. 20	850 µm
No. 25	710 µm
No. 30	600 µm
No. 35	500 µm
No. 40	425 µm
No. 50	300 µm
No. 60	250 µm
No. 70	212 µm
No. 80	180 µm
No. 100	150 µm
No. 120	125 µm
No. 140	106 µm
No. 170	90 µm
No. 200	75 µm
No. 230	66 µm
No. 270	53 µm
No. 325	45 µm
No. 400	38 µm

This is to say that this method of showing particle size distributions shouldn't have any barrier beyond simple statistical analysis software, a set of sieves, and a scale. The accessibility of this procedure is important as it is a universal problem. The scientific community unfortunately is sitting on tons of particle size data with very few effective ways to analyze multiple distributions at once. This would immediately allow the scientific community to look at and study particle size in a novel way. Conclusions can be drawn from many distributions at once, such as the hypothesis that a native Z modulus exists as a base property of a powder, or even that there are different inclusions in an analyzed powder resulting indicated through a shift in Z modulus.

II. LITERATURE REVIEW

Particle size distributions are a commonly collected feature when analyzing materials, especially ceramic materials. The D_{50} or the mean particle size is a characteristic often used to describe particle size distributions. It is the particle diameter found where half of the particles analyzed are larger and half of the particles are smaller. In industry this value is used as an index to describe distributions.[1]

In this study all historical datasets were obtained from an x-ray sedimentation instrument that analyzes particle size by the principles of Stokes' law. Specifically, that the equilibrium velocity of a particle through a medium of known viscosity from gravity is directly related to the size of the particle.[2] This method of determining particle size assumes that the particles are spherical.

Log-normal distributions are described by most particle size analysis guides as the favorite to best describe datasets from size measurement instruments.[3] In 1913 Hazen proposed that a probability axis would provide a linear plot.[4] This however was illustrated using a generated dataset rather than a measured particle size distribution leading to the subsequent publications using only data from 30% to 70% of the CMFT%.[2]

Z is globally accepted designation for a single standard deviation.[5] This is calculated and plotted to provide a probability axis that allows for a linear approximation in the program used for statistical analysis. This value is directly determined from the CMFT% percent in this study.

In a previous study Kini concluded that there are only three ways a particle size distribution can be described statistically: as log-normal, Weibull, or neither.[3] Her work proposed that the type of distribution was coupled to the presence or absence of cleavage planes in the crystal. In a subsequent study Decker concluded that Weibull distributions are more accurate in the description of particle size distributions obtained directly from comminution.[6] These studies however, both make steps towards the graphical representation of particle size distributions linearly which allow for the creation of a two-parameter model to show multiple distributions in a single plot.

III. EXPERIMENTAL PROCEUDURE

Historical particle size distribution datasets were obtained from the x-ray sedimentation instrument (Sedigraph 5100, Micromeritics, Inc., Norcross, GA). The x-ray sedimentation instrument measures the interference with an x-ray beam caused by suspended particles in a fluid of known properties as they settle as described by Stokes. This interference indicates the particle size distribution of the powder sample that is then reported by the instrument. The measurement of particle size in this way assumes an equivalent particle diameter. In this study 88 historical datasets describing 5 different materials were statistically analyzed using Excel (Excel Office 365, Microsoft Corporation, Redmond, WA). A list of materials analyzed is presented in Table I.

In order to represent particle size distributions as a log-normal plot in Excel the resulting data from the dataset must be given a Z value from an Excel function in order to accurately represent a probability axis. The Z value is the normal standard deviation from the mean. For example, the value 0 is equivalent to 50%. Notable values for Z are shown in Table II. To obtain a Z value the CMFT%, the percentage of mass below the measured particle diameter, is gathered from the x-ray sedimentation instrument must be converted using the function***, NORM.S.INV()***, in Excel. The function uses CMFT%, i.e. ***NORM.S.INV(CMFT%),*** values as a decimal with the assumption of a normal distribution and assigns a Z value based on the distance from 50%. The application of this function is shown in the datasets provided digitally for future analysis (attached in a CD-format as an appendix). The example application of this function is shown in Table III. After the Z values are obtained in relation to a particular particle diameter reported by the x-ray sedimentation instrument, a plot can be produced that will offer a linear approximation of the particle size distribution resulting in two parameters for plotting, the D_{50}, and the slope, or Z modulus. An example of the plot is shown in Figure 1 and the equation for the Z modulus of a log-normal distribution is shown in Equation 1. The two parameters can then be plotted in comparison to other particle size distributions for similar and different materials.

Similarly, the particle size distributions can be represented as Weibull plots resulting in the parameters analyzed being the D_{50} and the Weibull modulus.

Table II. Powders analyzed from historical datasets.

Powder	**Frequency**
Alumina	45
Cerium Oxide	2
Glass Frit	7
Quartz	22
Silicon Carbide	10
Total	86

Table III. Notable values for the description of Z

Z	Std. Dev.	%	Overall %
0	Mean	-	-
±1	1	34.1	68.2
±2	2	47.7	95.4
±3	3	49.8	99.6

Table IV. The use of Excel to obtain Z. Highlighted is Z=0, identifying the D_{50} as the particle size at 50% or the mean of a log-normal distribution.

Particle Size (µm)	CMFT%	Z NORM.S.INV(CMFT%)
8.660	0.245	-0.691
9.173	0.299	-0.527
9.716	0.358	-0.364
10.292	0.420	-0.201
10.902	0.485	-0.038
11.050	0.500	0.000
11.548	0.549	0.124
12.232	0.613	0.287
12.957	0.673	0.449
13.725	0.730	0.611
14.538	0.780	0.774

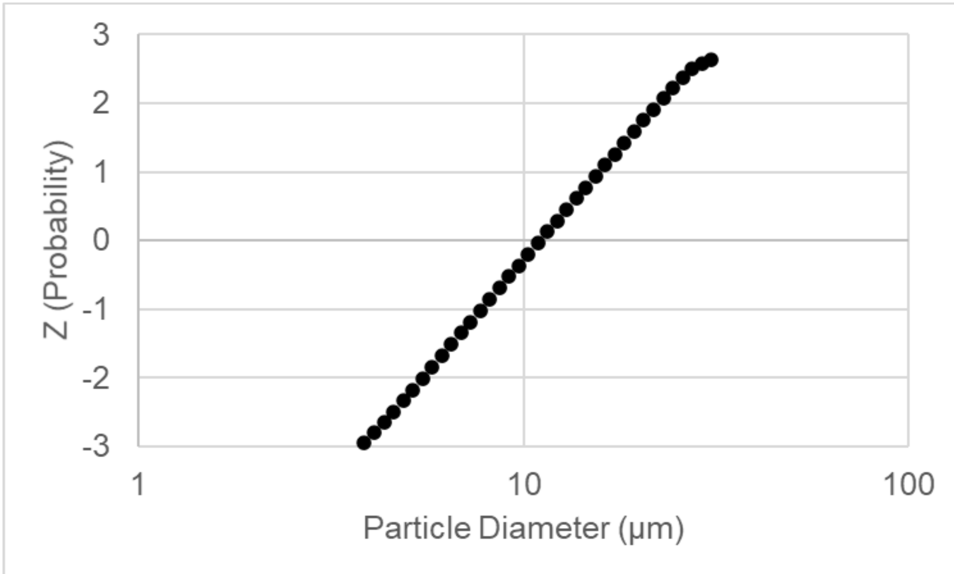

Figure 2. The particle size distribution of a sample of silicon carbide represented as a log-normal distribution.

$$Z\ modulus = \frac{\Delta Z}{\Delta PD} \rightarrow \frac{\Delta Z}{\Delta \log{(PD)}} \tag{1}$$

IV. RESULTS AND DISCUSSION

4.1 Log-Normal Analysis

In order to analyze the powders using log-normal distributions it is important to illustrate the plots that are used and where it is derived from. Figure 2 through Figure 9 show examples of CMFT% plotted as a function of log particle diameter and Z as a function of log particle diameter for randomly selected examples of representative powder datasets for each powder type.

In general terms, the slope of Z versus log-particle size, is termed the "Z-modulus." As the Z-modulus increases, the width of the distribution becomes narrower, analogous to the Weibull modulus when describing brittle failure strength data.

For this analysis, the data is assumed to fit a log-normal distribution if the regression coefficient, R^2, is greater than 0.75. Datasets that did not meet this criterion were then evaluated using a Weibull analysis, i.e., a skewed log-normal distribution.

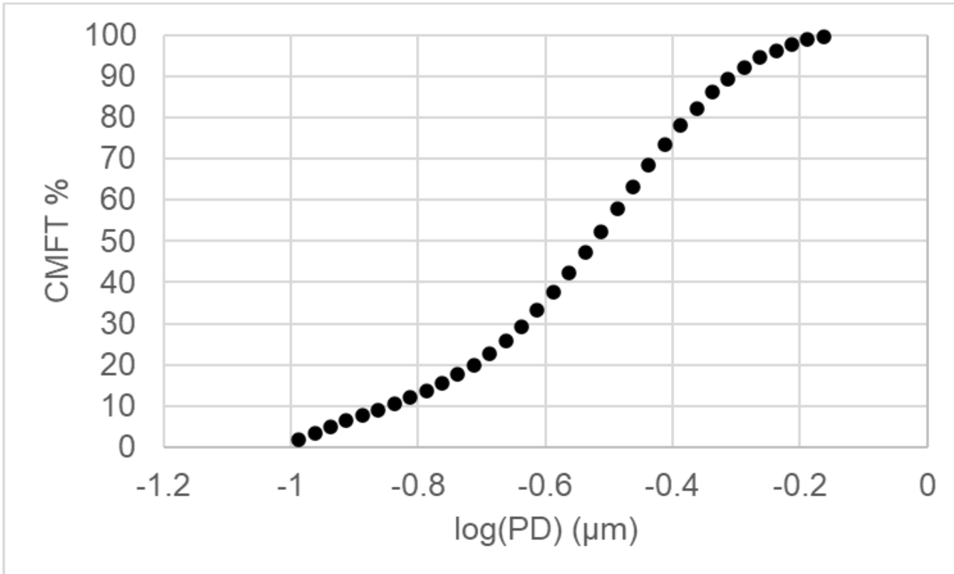

Figure 3. A calcined alumina dataset represented as CMFT% as a function of log particle diameter.

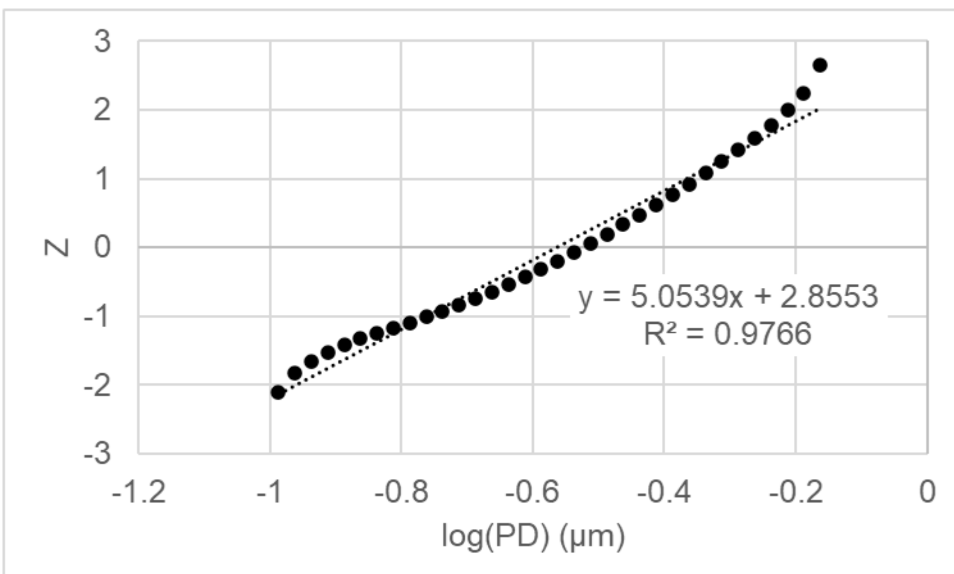

Figure 4. A calcined alumina dataset represented as Z as a function of log particle diameter.

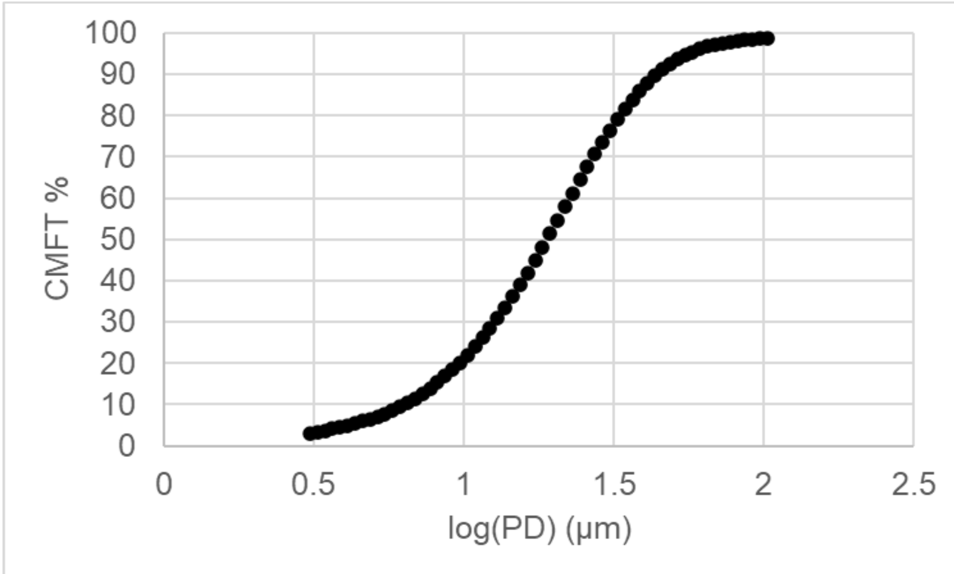

Figure 5. A glass frit dataset represented as CMFT% as a function of log particle diameter.

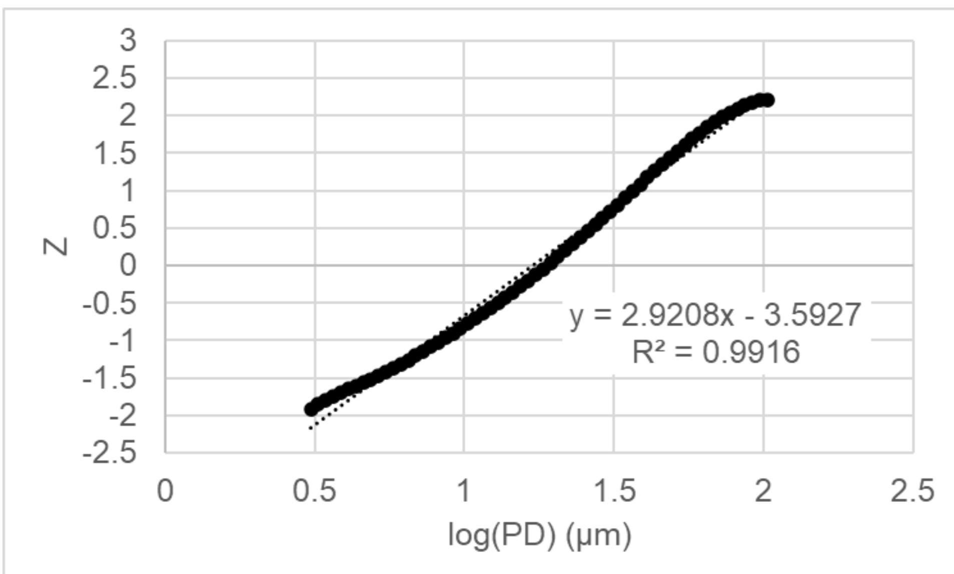

Figure 6. A glass frit dataset represented as Z as a function of log particle diameter.

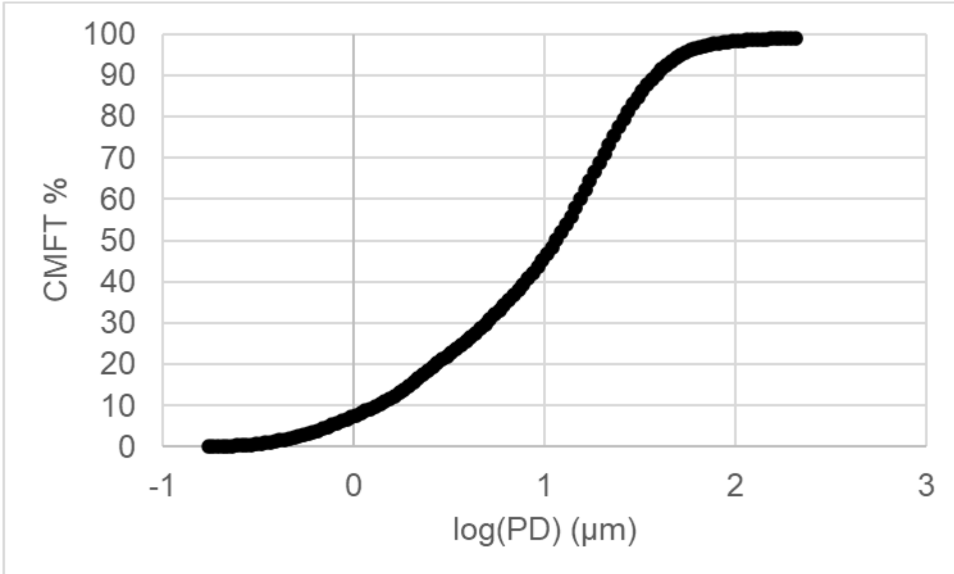

Figure 7. A quartz dataset represented as CMFT% as a function of log particle diameter.

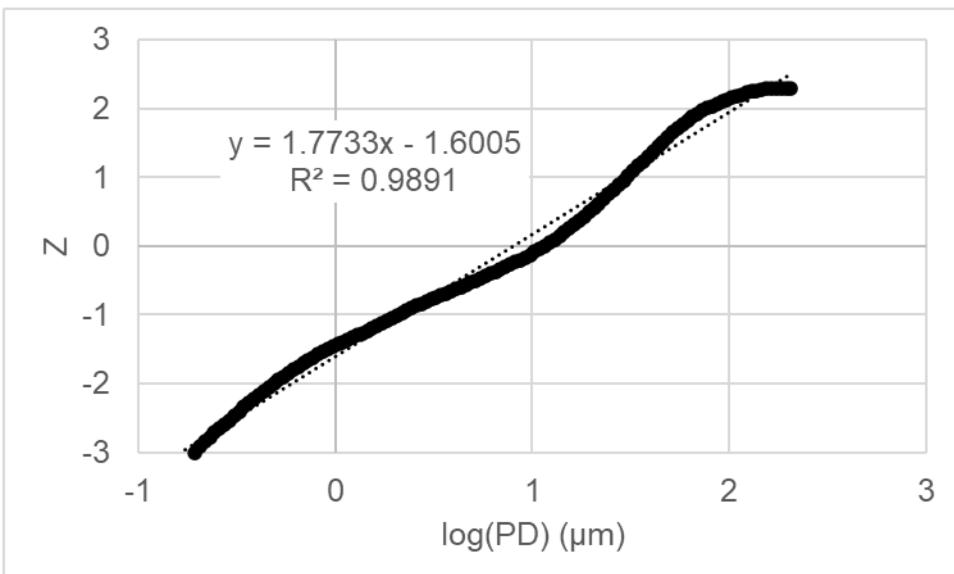

Figure 8. A quartz dataset represented as Z as a function of log particle diameter.

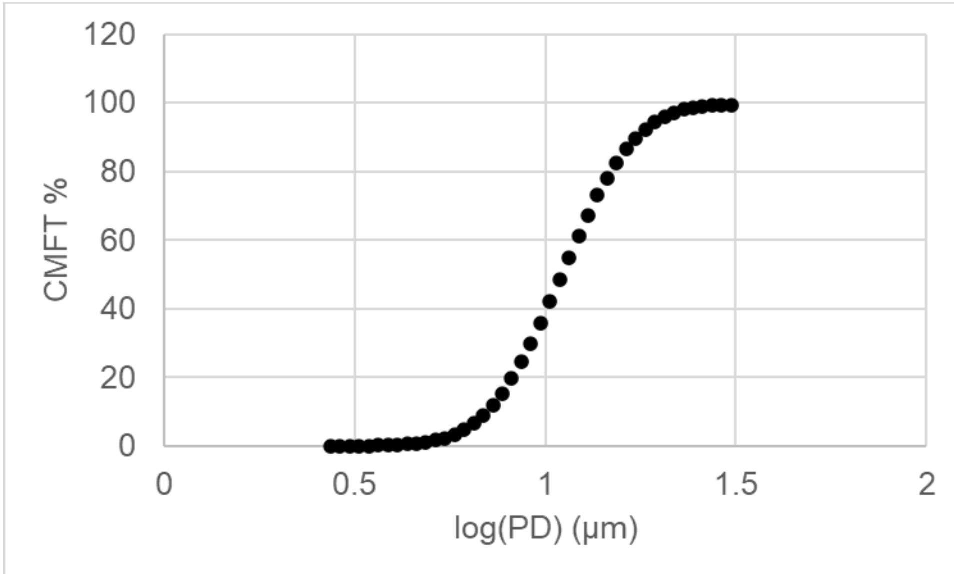

Figure 9. A silicon carbide dataset represented as CMFT% as a function of log particle diameter.

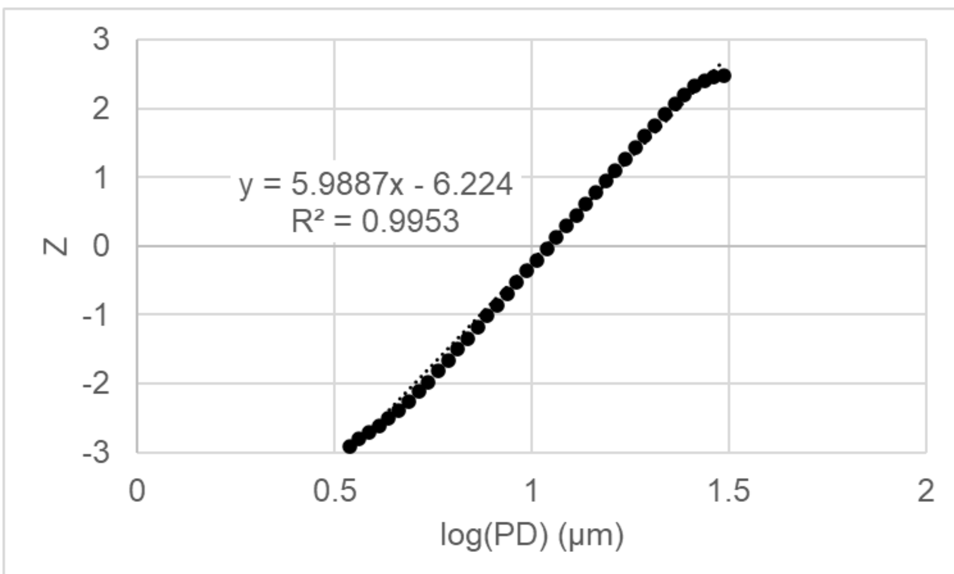

Figure 10. A silicon carbide dataset represented as Z as a function of log particle diameter.

Following the procedure to convert CMFT% into a Z value the D_{50} and Z modulus can be gathered for every dataset evaluated. Table IV through Table Xi show the results for the representative powder's datasets. In the tables the value for R^2 or the coefficient of determination is included, this value indicates the amount of linear association between the two variables. In this case the variables being evaluated to show the R^2 value is the Z value and log particle diameter.

Table V. Table of results for A3000 alumina for log-normal analysis.

D_{50} (μm)	**Z Modulus**	**R^2**
0.273	4.3862	0.9579
0.299	5.0539	0.9766
0.322	4.3683	0.9827
0.326	4.9408	0.9810
0.326	3.6910	0.9781
0.328	4.4846	0.9899
0.329	4.6021	0.9915
0.339	4.6675	0.9907
0.355	4.8357	0.9693
0.383	3.9910	0.9928
0.406	4.3453	0.9944
0.425	3.7012	0.9954
0.436	3.9189	0.9957
0.813	3.3445	0.9180

Table VI. Table of results for A-16 S.G. alumina for log-normal analysis.

D_{50} (μm)	Z Modulus	R^2
0.311	2.1507	0.8312
0.323	1.1648	0.7594
0.330	1.7552	0.7829
0.331	1.5299	0.8314
0.337	1.5227	0.8269
0.350	1.6232	0.7776
0.365	1.6423	0.9318
0.370	1.4082	0.8170
0.422	1.6314	0.8867
0.423	1.6114	0.8801
0.425	1.5609	0.8913
0.445	1.8305	0.9116
0.448	1.2979	0.8539
0.449	1.9150	0.9211
0.453	1.5485	0.9063
0.466	1.3931	0.8843
0.470	1.1637	0.8600
0.532	1.3862	0.8882
0.539	1.6807	0.9381
0.572	1.9767	0.9514
0.573	1.2020	0.7656
0.593	1.4103	0.9268
0.597	2.0511	0.9481
0.651	1.2344	0.8431
0.876	1.6502	0.9074

Table VII. Table of results for A-10 alumina for log-normal analysis.

D_{50} (μm)	Z Modulus	R^2
2.448	1.6679	0.9702
3.511	1.3719	0.9260
5.501	1.2038	0.8297

Table VIII. Table of results for tabular alumina for log-normal analysis.

D_{50} (µm)	Z Modulus	R^2
7.234	1.4933	0.8616
7.367	1.9940	0.9618
8.937	1.6323	0.9954

Table IX. Table of results for glass frit for log-normal analysis.

D_{50} (µm)	Z Modulus	R^2
18.17	2.7503	0.9807
18.70	3.2066	0.9918
18.91	2.9208	0.9916
20.24	3.2329	0.9794
20.59	2.7190	0.9883
21.51	3.1392	0.9730
22.89	3.1670	0.9873

Table X. Table of results for quartz for log-normal analysis.

D_{50} (µm)	Z Modulus	R^2
7.720	1.9734	0.9521
7.726	1.6861	0.9775
7.906	1.7251	0.9532
8.727	2.0221	0.9505
8.738	2.1574	0.9514
8.867	2.1595	0.9463
11.42	1.8127	0.9587
11.51	1.7733	0.9891
11.56	1.7460	0.9653
13.21	1.7317	0.9359
13.23	1.8437	0.9424
13.37	1.8992	0.9868
13.39	1.8430	0.9450
13.39	1.7992	0.9482
13.56	1.7112	0.9713
13.56	2.3059	0.9742
13.60	2.1990	0.9832
17.79	1.8948	0.9855
20.89	1.6794	0.9402
27.13	2.0218	0.9365
27.15	1.8764	0.9697
28.04	1.8322	0.9722

Table XI. Table of results for silicon carbide for log-normal analysis.

D_{50} (µm)	Z Modulus	R^2
10.45	6.1782	0.9919
11.00	5.9046	0.9883
11.04	4.5057	0.9820
11.05	6.3984	0.9993
11.05	5.9887	0.9953
20.85	4.3466	0.9379
20.89	4.8555	0.9863
21.05	5.4100	0.9923
21.19	5.2444	0.9930
21.40	4.7065	0.9689

Table XII Table of results for cerium oxide for log-normal analysis.

D_{50} (µm)	Z Modulus	R^2
0.642	1.8386	0.9881
1.396	1.9693	0.9695

Particle size distributions plotted as a log-normal plot yield two parameters, a D_{50} and a Z modulus. These parameters can be used to create a D_{50} vs Z modulus plot to compare multiple particle size distributions at once. This plot is shown in Figure 10. The square symbols in Figure 10 are alumina datasets. Note that Figure 10 does not include the two datasets for cerium oxide as there are not enough points to adequately compare distributions.

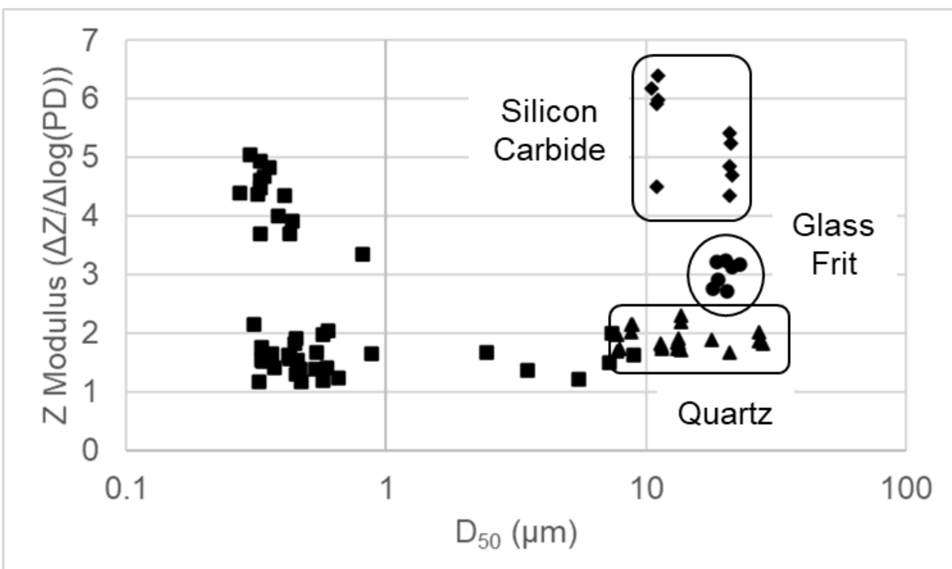

Figure 11. Plot of compiled particle size distributions for various materials.

The plot is labeled for the materials silicon carbide, glass frit, and quartz. The alumina datasets are unlabeled in Figure 10. The compilation of the alumina particle size distributions is shown in Figure 11 showing that the type of alumina appears to contribute to particle size distribution data.

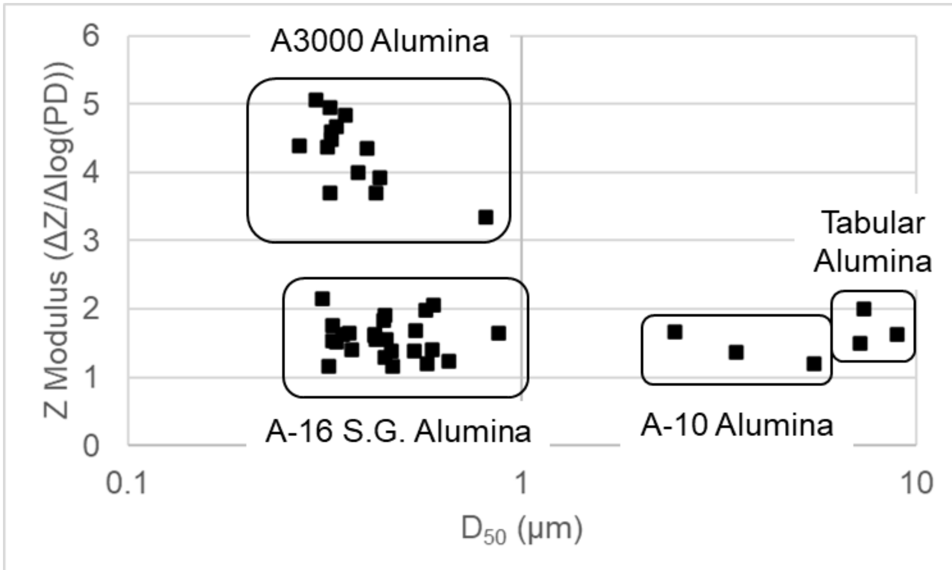

Figure 12. Plot of compiled distributions for alumina powders.

In Figure 10 it is evident that in every case except with silicon carbide the Z modulus for each material remains relatively tightly grouped. The D_{50} for the quartz datasets changes throughout the course of the historical measurements. This is likely to be evidence of a milling study in which a native distribution of quartz is milled reducing its D_{50} but retaining its distribution features. This is strongly suggests that a given material will exhibit a native Z modulus providing an opportunity to detect contamination or changes to the material, potentially through processing variations in the fabrication of the powder.

Supporting evidence for this hypothesis can be gathered from the silicon carbide datasets plotted together. The reduction of the slope suggests the distribution widening. This can be done through milling similarly to the quartz samples. The increase in frequency of the Z-modulus of silicon carbide at about 4.5 may suggest a native Z-modulus exists there as well. Figure 3 also could make an argument for the existence of a native Z-modulus for alumina powders at a Z-modulus of around 1.5. The function of the Z-modulus as an indicator for behavior of the particle size distribution is also on display in Figure 3. Both A3000 alumina and A-16 S.G. alumina are calcined aluminas. The similarity of the D_{50} yet different Z-modulus suggest that A3000 alumina is produced with, or scalped to, a significantly narrower distribution than A-16 S.G. alumina.

4.2 Weibull Analysis

In order to analyze the powders using Weibull distribution it is important to illustrate the plots that are used and where it is derived from. To obtain a Weibull distribution Equation 2 is applied to the CMFT% as a decimal to produce the values for a Weibull axis. The values for a Weibull axis are plotted as a function of natural log particle diameter to produce the Weibull plot. Figure 12 through Figure 20 show an example CMFT vs natural log particle diameter plot and Weibull plots for representative powders.

$$\ln\left(-\ln(CMFT\%)\right) \qquad (2)$$

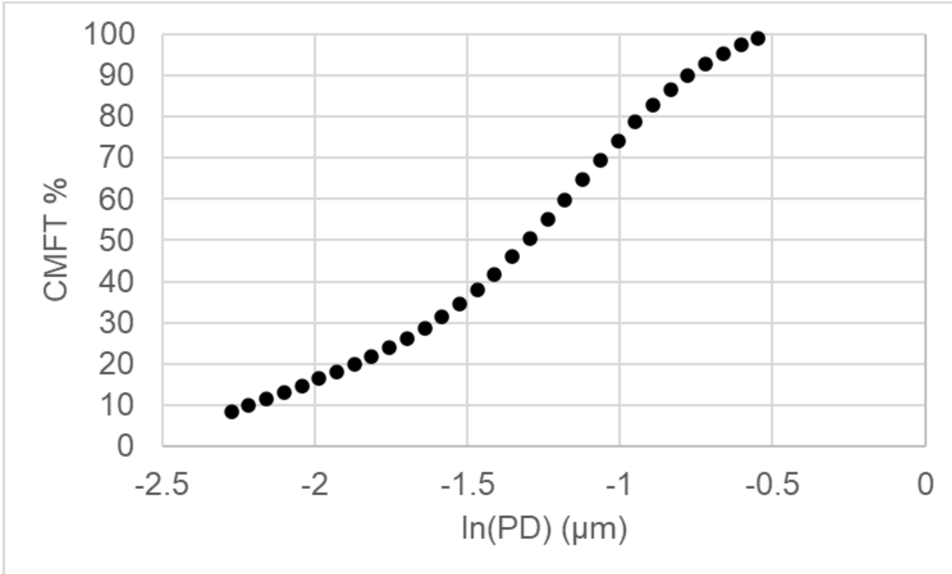

Figure 13. A calcined alumina dataset represented as CMFT% as a function of natural log particle diameter.

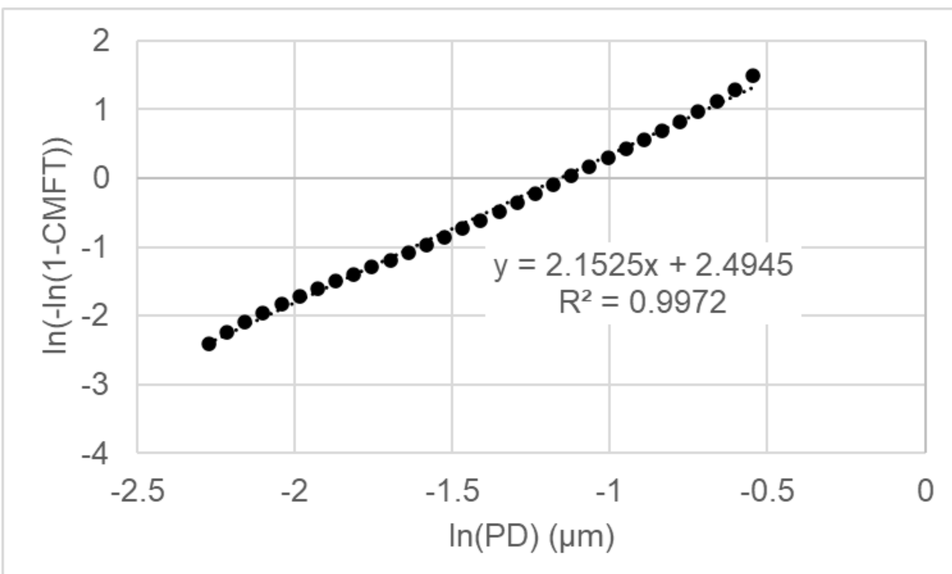

Figure 14. A calcined alumina dataset represented as a Weibull plot.

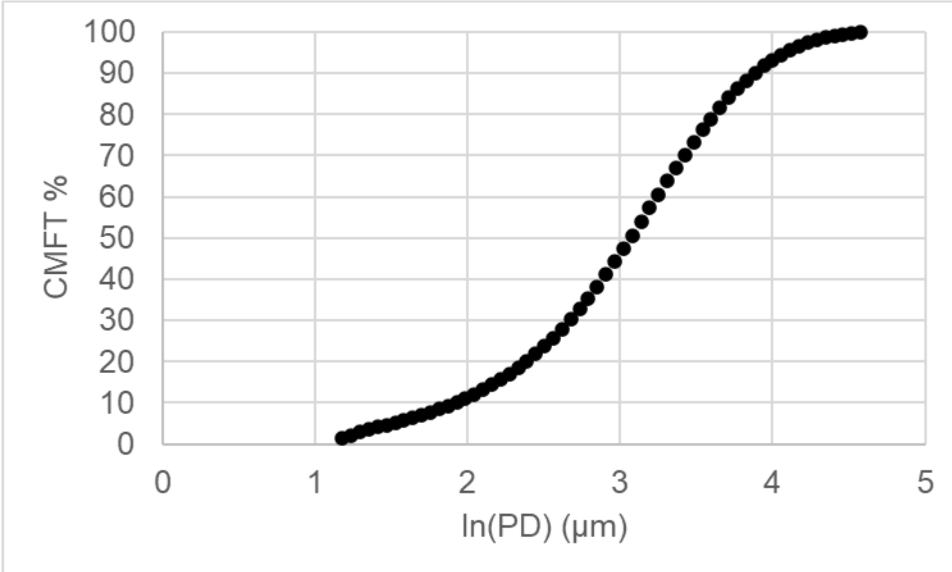

Figure 15. A glass frit dataset represented as CMFT% as a function of natural log particle diameter.

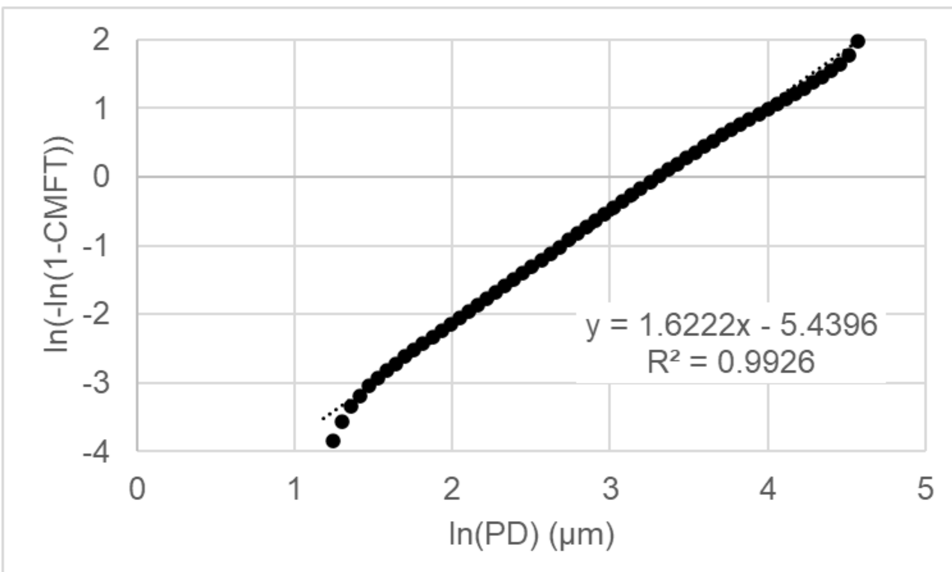

Figure 16. A glass frit dataset represented as a Weibull plot.

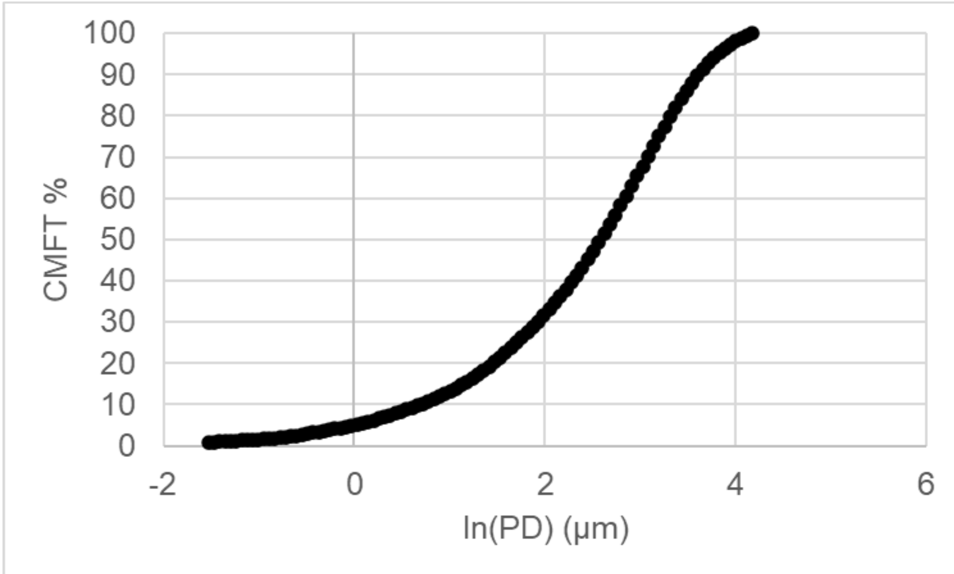

Figure 17. A quartz dataset represented as CMFT% as a function of natural log particle diameter.

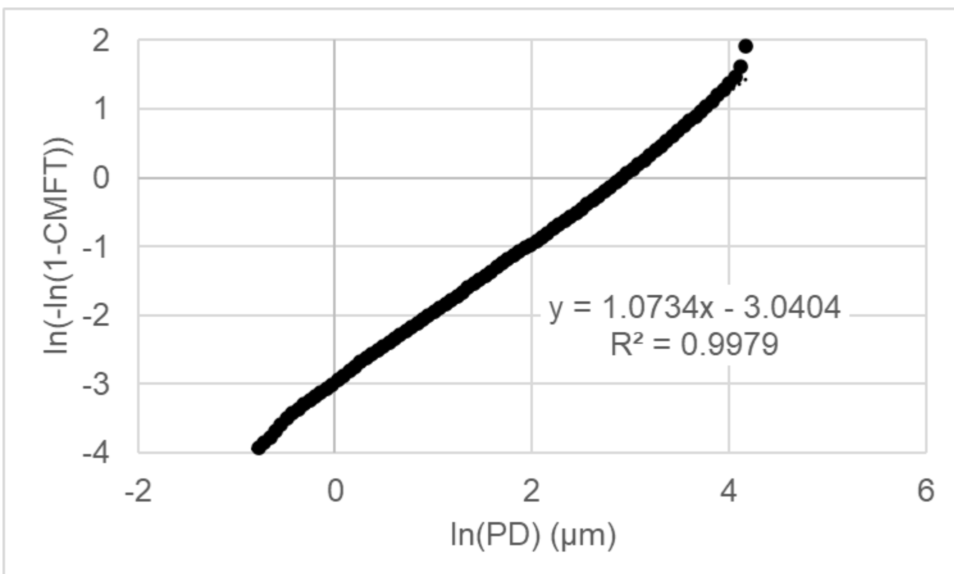

Figure 18. A quartz dataset represented as a Weibull plot.

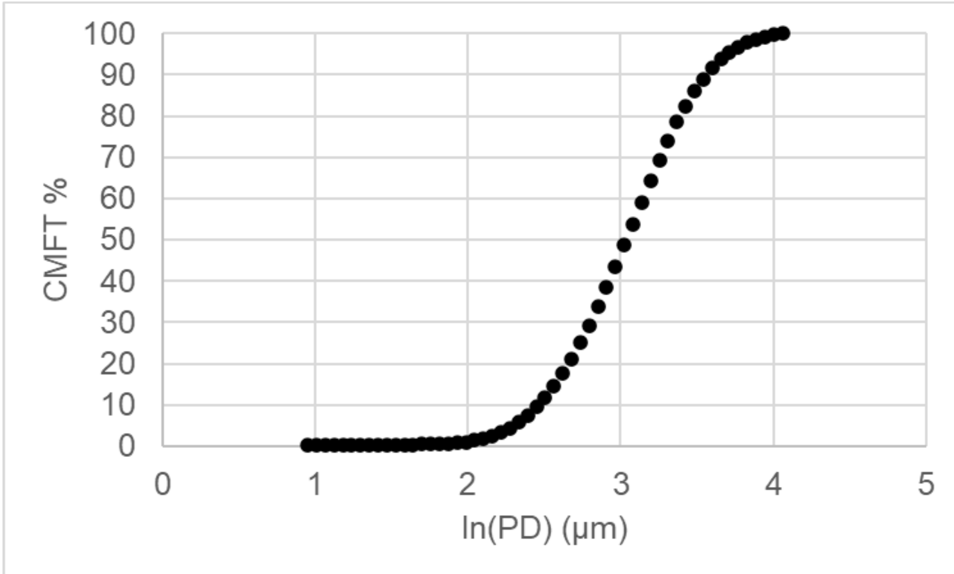

Figure 19. A silicon carbide dataset represented as CMFT% as a function of natural log particle diameter.

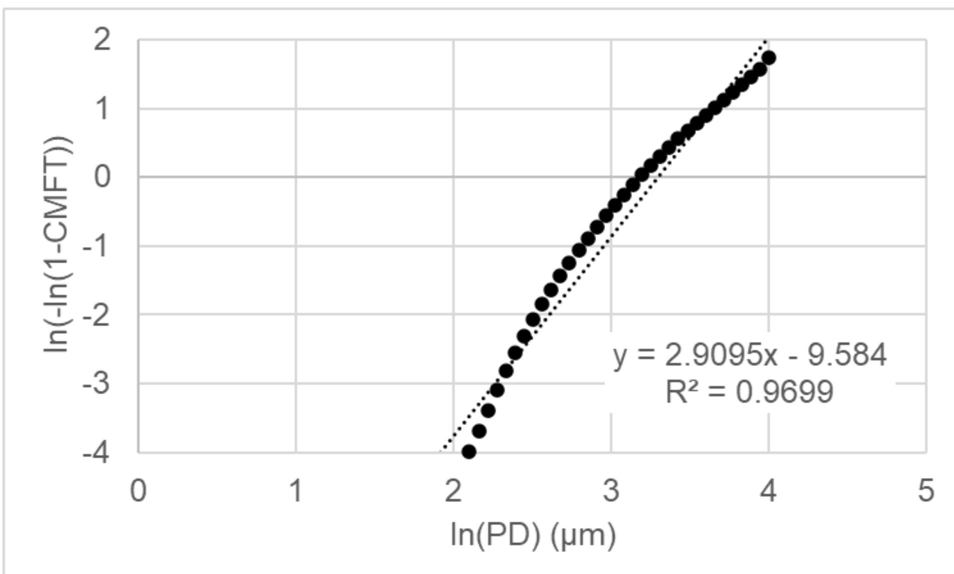

Figure 20. A silicon carbide dataset represented as a Weibull plot.

Converting CMFT% into values for the Weibull axis, the D_{50} and slope can be gathered for every dataset evaluated. Table XII through Table XIX show the results for each representative powder's datasets. In this case the R^2 value is the result of comparing the relationship between the Weibull value and natural log particle diameter.

Table XIII. Table of results for A3000 alumina for Weibull analysis.

D_{50} (µm)	Weibull Modulus	R^2
0.273	2.1525	0.9972
0.299	2.6311	0.9882
0.322	1.9196	0.9943
0.326	2.4455	0.9938
0.326	1.8344	0.9405
0.328	1.8945	0.9483
0.329	2.1729	0.9815
0.339	2.2510	0.9821
0.355	2.4167	0.9942
0.383	1.5729	0.9667
0.406	2.2141	0.9731
0.425	1.8112	0.9622
0.436	1.8593	0.9646
0.813	1.8053	0.8646

Table XIV. Table of results for A-16 S.G. alumina for Weibull analysis.

D_{50} (μm)	Weibull Modulus	R^2
0.311	0.9266	0.7357
0.323	0.4963	0.6302
0.330	0.8043	0.6654
0.331	0.6515	0.7262
0.337	0.6646	0.7185
0.350	0.8101	0.7177
0.365	0.6256	0.8390
0.370	0.6172	0.6919
0.422	0.7261	0.7719
0.423	0.7177	0.7531
0.425	0.6919	0.7664
0.445	0.8793	0.8188
0.448	0.5901	0.7247
0.449	0.8913	0.8255
0.453	0.6820	0.7851
0.466	0.6291	0.7511
0.470	0.5157	0.7228
0.532	0.6413	0.7672
0.539	0.8309	0.7955
0.572	0.9362	0.8493
0.573	0.7737	0.6773
0.593	0.6278	0.8073
0.597	0.9419	0.8420
0.651	0.6493	0.7341
0.876	0.9474	0.8126

Table XV. Table of results for A-10 alumina for Weibull analysis.

D_{50} (μm)	Weibull Modulus	R^2
2.448	0.7284	0.9013
3.511	0.6706	0.8760
5.501	0.7225	0.8277

Table XVI. Table of results for tabular alumina for Weibull analysis.

D_{50} (µm)	Weibull Modulus	R^2
7.234	0.7870	0.8897
7.367	1.1651	0.9922
8.937	1.0673	0.9561

Table XVII. Table of results for glass frit for Weibull analysis.

D_{50} (µm)	Weibull Modulus	R^2
18.17	1.2815	0.9457
18.70	1.4419	0.9819
18.91	1.4998	0.9871
20.24	1.6540	0.9951
20.59	1.2955	0.9620
21.51	1.6222	0.9926
22.89	1.5065	0.9863

Table XVIII. Table of results for quartz for Weibull analysis.

D_{50} (μm)	Weibull Modulus	R^2
7.720	1.1097	0.9927
7.726	0.9526	0.9682
7.906	1.0640	0.9978
8.727	1.1852	0.9815
8.738	1.2140	0.9894
8.867	1.2333	0.9822
11.42	1.0903	0.9694
11.51	1.0618	0.9311
11.56	1.1193	0.9033
13.21	1.0734	0.9979
13.23	1.1107	0.9955
13.37	1.1031	0.9438
13.39	1.0497	0.9219
13.39	1.0571	0.9396
13.56	1.0019	0.9380
13.56	1.3365	0.9458
13.60	1.2581	0.9486
17.79	1.3861	0.9729
20.89	1.2194	0.9906
27.13	1.2588	0.9496
27.15	1.1591	0.9867
28.04	1.137	0.9662

Table XIX. Table of results for silicon carbide for Weibull analysis.

D_{50} (μm)	Weibull Modulus	R^2
10.45	2.5731	0.9207
11.00	3.7198	0.8799
11.04	2.5202	0.9603
11.05	4.0887	0.9316
11.05	3.9542	0.9555
20.85	2.9095	0.9699
20.89	2.7325	0.9323
21.05	3.2796	0.9616
21.19	2.9506	0.8985
21.40	2.2523	0.9292

Table XX. Table of results for cerium oxide for Weibull analysis.

D_{50} (µm)	Z Modulus	R^2
0.642	0.8084	0.9693
1.396	0.9664	0.9978

Particle size distributions plotted as a Weibull plot yield two parameters, a D_{50} and a Weibull modulus. These parameters can be used to create a D_{50} vs Weibull modulus plot to compare multiple particle size distributions at once. This plot is shown in Figure 20 (omitting the CeO_2 datasets).

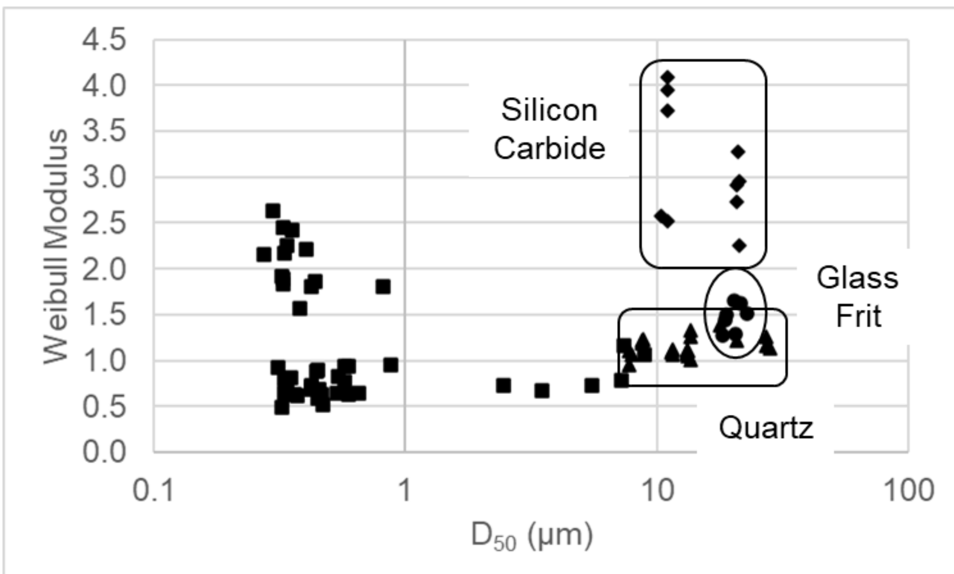

Figure 21. Plot of compiled particle size distributions for various materials using Weibull analysis.

The plot is labeled for the materials silicon carbide, glass frit, and quartz. The alumina datasets are unlabeled in Figure 20. The compilation of the alumina particle size distributions is shown in Figure 21.

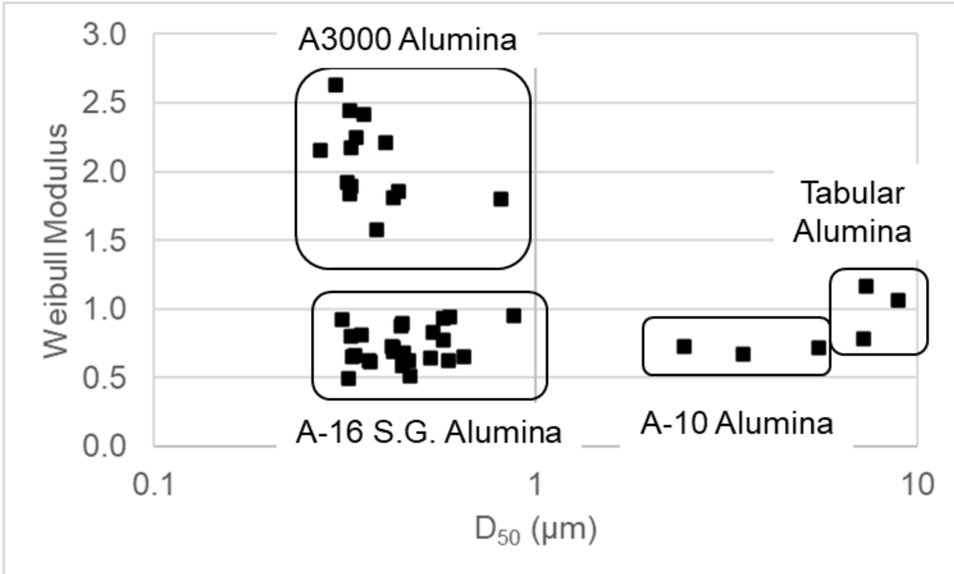

Figure 22. Plot of compiled distributions for alumina powders using Weibull analysis.

The plots in Figures 20 and 21 illustrate similar properties as those shown using log-normal plots to show the datasets. This suggests both methods of showing particle size distributions are capable of showing the data with relatively similar results.

4.3 Scalped Distributions

It is a trend that native distributions tend to fit Weibull distributions. Figure 22 shows the linear approximation of a distribution of quartz that fits a Weibull distribution better than a log-normal distribution. This is determined by their R^2 value which indicates how well the variables relate in a linear approximation.

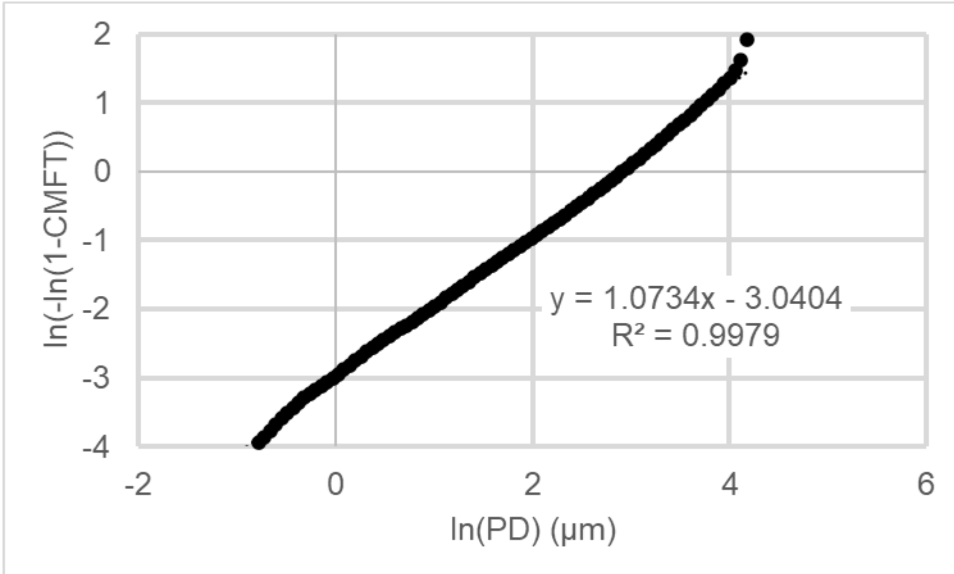

Figure 23. A quartz dataset plotted using Weibull distribution to find a linear approximation.

The same particle size dataset for quartz is shown in Figure 23 as a log-normal distribution. The linear approximation is relatively poor when compared to the fit of the Weibull plot.

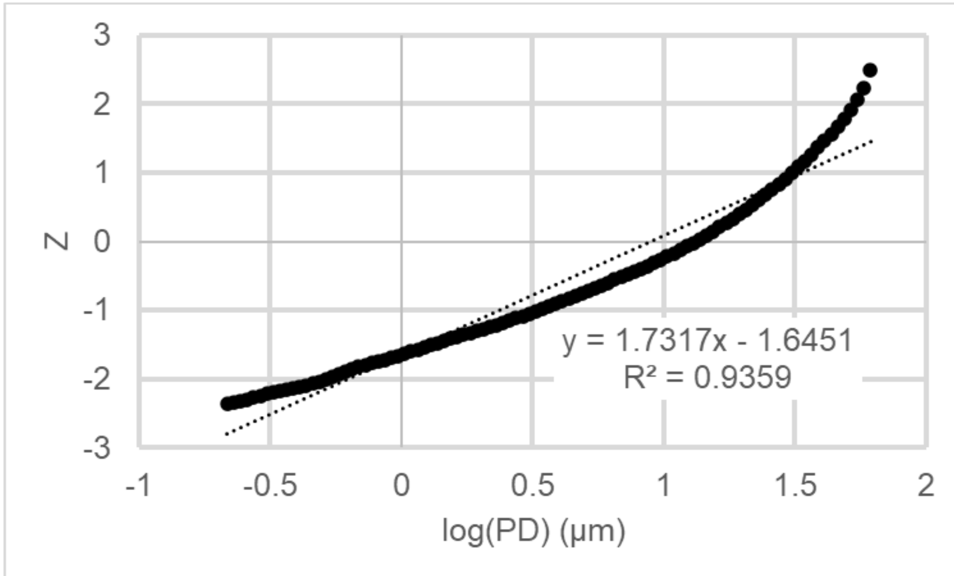

Figure 24. A quartz dataset plotted using log-normal distribution to find a linear approximation.

This trend likely means that by scalping the same distribution a better log-normal linear approximation should be achievable. In Figure 24 the same dataset is scalped, removing particles smaller than 3.0 μm and larger than 30 μm to simulate industrial scalping (based on particle size). The remaining data is then renormalized and evaluated using log-normal distribution.

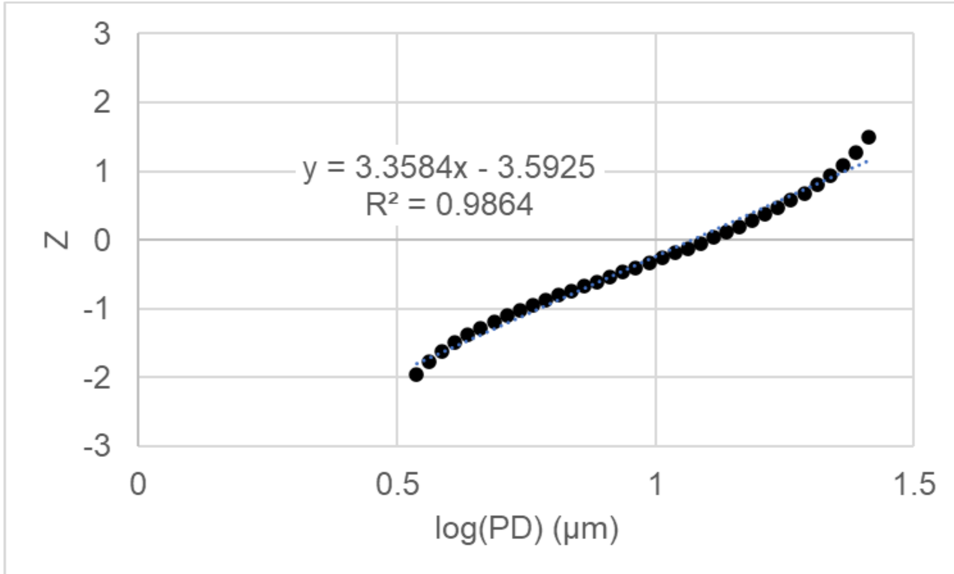

Figure 25. A scalped quartz dataset plotted using log-normal distribution to find a linear approximation.

Figure 25 shows the same scalped dataset represented using a Weibull plot to find a linear approximation.

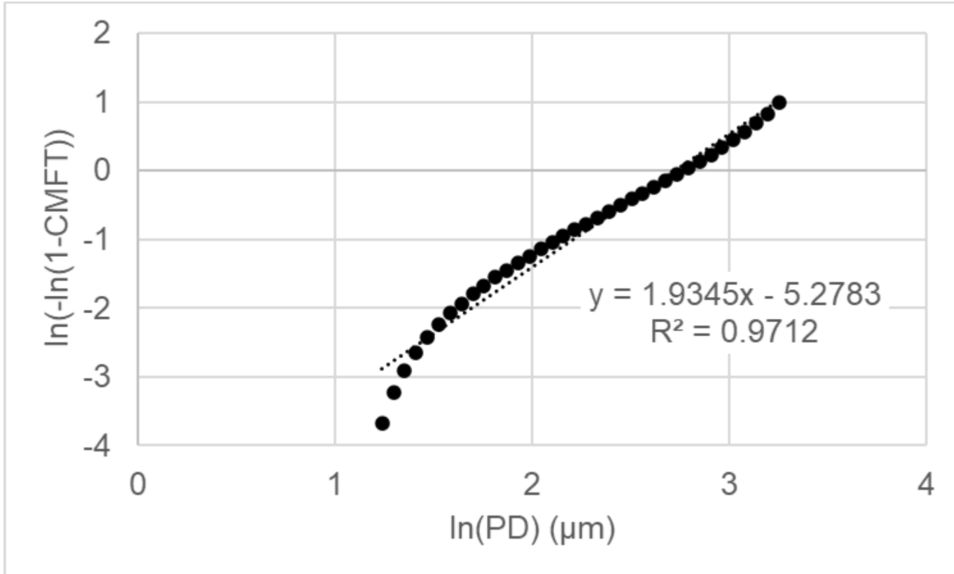

Figure 26. A scalped quartz dataset plotted as a Weibull distribution illustrating that the fit is poorer than observed for log-normal (in Figure 24).

This difference in fit for the linear approximation would suggest that for the same dataset a native distribution is best represented as a Weibull distribution, and a scalped distribution is best represented using log-normal analysis to determine the Z modulus. The difference in Z modulus and Weibull modulus between the native and scalped distributions is indicative of the properties shown by the respective modulus, i.e. a higher modulus indicates a narrower distribution.

V. SUMMARY AND CONCLUSIONS

The particle size distribution of various materials can be represented as a two-parameter model in order to compare and analyze multiple distributions at once. This can be done using log-normal or Weibull distributions. This study suggests the existence of native particle size distribution and a native Z modulus. The use of a two-parameter model to describe particle size distributions can be used by industry for quality control applications and facilitate tracking of particle size distributions of incoming raw materials. Furthermore, it may be useful to analyze and track particle size distribution changes during a milling study. Further work is necessary to build a catalogue of raw material distributions to test the hypothesis of a native particle size distribution that remains consistent despite changing D_{50} values.

VI. REFERENCES

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